A survey on moment problems

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Given a closed subset $K \subset R^n$, the K-moment problem concerns conditions on a real sequence $\beta \equiv \{\beta_i\}_{i \in Z_+^n}$, so that there exists a positive Borel measure μ with closed support *supp* $\mu \subset K$ such that $\beta_i = \int x^i d\mu(x)$ $(i \in Z_+^n)$. The Riesz functional Λ associates to a polynomial $p(x) \equiv \sum a_i x^i$ the value $\sum a_i \beta_i$. The classical Riesz-Haviland theorem shows that there exists a K-representing measure for β (as above) if and only if $p|K \geq 0$ implies $\Lambda(p) \geq 0$. K. Schmüdgen (1991) found a concrete solution in the case when K is compact and semialgebraic, and Schmüdgen's result has recently been extended is several directions.

In joint work with R.E. Curto, we focus on the semialgebraic and algebraic cases of the *truncated* K-moment problem, where the moment data is restricted so that $|i| \leq 2d$ and the moment data $\beta^{(2d)}$ thus correspond to a moment matrix $M(d)$. A result of J. Stochel shows that solubility of the truncated K-moment problem for each d is equivalent to solubility of the full K-moment problem. The existence of K-representing measures for $\beta^{(2d)}$ can be studied in terms of positivity and extension properties of $M(d)$; a complete solution is known for the case when K is a planar curve of degree two.