

**Additional Exercise for the LNMB course CO1a**  
**Monday October 4, 2010**

Let  $G = (V, E)$  be a bipartite graph and  $w \in \mathfrak{R}^E$  be weights on the edges.

1) Let  $M$  and  $N$  be two matchings with  $|N| = |M| + 2$ . Show that one can find two matchings  $M_1$  and  $M_2$  such that

$$M \cup N = M_1 \cup M_2, \quad M \cap N = M_1 \cap M_2, \quad |M_1| = |M_2| = |M| + 1.$$

2) Let  $W_k$  denote the maximum weight of a matching of cardinality  $k$ . Show that

$$W_k + W_{k+2} \leq 2W_{k+1}.$$

3) Show that in the weighted bipartite matching algorithm, one can stop as soon as one finds an extreme matching  $M$ , a minimum length  $M$ -augmenting path  $P$  such that  $w(M \Delta P) \leq w(M)$ .