LNMB - NETWORKS AND SEMIDEFINITE PROGRAMMING
EXERCISE - LECTURE DECEMBER 3, 2012

Exercise. Let $G = (V = [n], E)$ be a graph and let $\mathcal{C}_G$ denote the collection of cliques of G. Then define the polytope:

$$\mathrm{QST}(G) = \{x \in \mathbb{R}^n : x \geq 0,\ \sum_{i \in C} x_i \leq 1\ \forall C \in \mathcal{C}_G\}.$$

Given nonnegative node weights $w \in \mathbb{R}^n_+$, $\alpha(G, w)$ denote the maximum weight of a stable set in G and $\alpha^*(G, w)$ is defined by

$$\alpha^*(G, w) = \max\{w^T x : x \in \mathrm{QST}(G).$$

1) Show that $\alpha(G, w) \leq \alpha^*(G, w)$.
2) Give an instance (G, w) showing that the inequality in 1) can be strict.
3) Show that in the definition of $\alpha^*(G, w)$ one can omit the nonnegativity condition; that is,

$$\alpha^*(G, w) = \max\{w^T x : \sum_{i \in C} x_i \leq 1\ \forall C \in \mathcal{C}_G\}.$$

4) Show that

$$\alpha^*(G, w) = \min\{\sum_{C \in \mathcal{C}_G} \lambda_C : \sum_{C \in \mathcal{C}_G : i \in C} \lambda_C = w_i\ \forall i \in [n],\ \lambda_C \geq 0\ \forall C \in \mathcal{C}_G\}.$$