LNMB - NETWORKS AND SEMIDEFINITE PROGRAMMING

Lecture on January 28, 2018

Exercise. Given a graph $G = (V = [n], E)$, let $\overline{E}$ denote the set of pairs $\{i, j\} \subseteq V$ with $i \neq j$ and $\{i, j\} \notin E$. A symmetric $n \times n$ matrix B is said to *fit* G if $B_{ii} \neq 0$ for all $i \in V$ and $B_{ij} = 0$ for all $\{i, j\} \in \overline{E}$. Consider the parameter $R(G)$ defined as the smallest possible rank of a matrix B which fits G, i.e.,

$$R(G) = \min\{\text{rank}(B) : B_{ii} \neq 0 \text{ for } i \in V,\ B_{ij} = 0 \ \text{ for } \{i, j\} \in \overline{E}\}.$$

a. Show that $\alpha(G) \leq R(G) \leq \chi(\overline{G})$.

b. What is the value of $R(C_5)$?

c. Show that $R(G \boxtimes H) \leq R(G)R(H)$.

d. Show that $\Theta(G) \leq R(G)$.