

# Approximation hierarchies for (non-)commutative polynomial optimization

This PhD position is funded by the Marie Curie program of European Union through the innovative training network (ITN) POEMA on polynomial optimization.

More info and positions at <https://easychair.org/cfp/POEMA-19-22>.

Contact at CWI: Monique Laurent, [M.Laurent@cwi.nl](mailto:M.Laurent@cwi.nl)

**Scientific context.** Many fundamental problems in operations research, applied mathematics, control theory and quantum information can be modelled as instances of polynomial optimization problems. The versatility of this model lies in the choice of the domain of the variables, which can be scalars (binary or continuous), matrices (with fixed or arbitrary size) or operators, and it is captured by using polynomial algebra with non-commutative variables and problem specific prescribed commutativity rules. For instance, binary variables arise naturally in discrete optimization and Hermitian projections, their operator analogues, are typically used in quantum information, and this general model captures both eigenvalue and trace optimization problems. This project aims to investigate hierarchies of approximations for these general polynomial optimization problems. The methodology will rely on combining tools from real algebra, moment theory, operator theory, and semidefinite optimization. Examples of questions to be addressed include:

- Convergence analysis of the approximation hierarchies;
- Genericity of finite convergence and optimality certificates;
- Exploring techniques (like sparsity and using appropriate polynomial bases) to make the computation of the non-commutative hierarchies more efficient.

**Working Context.** The PhD candidate will be hosted by the research group *Networks and Optimization* at CWI (Centrum Wiskunde & Informatica). CWI is the Dutch national research institute for mathematics and computer science, located in Amsterdam. The group is internationally recognized for its expertise in algorithmic, algebraic and geometric methods for discrete and continuous optimization. The group, and CWI at large, hosts international students, postdocs and senior researchers in a nice and stimulating working environment. Next to possible collaborations with other CWI groups, the candidate will also benefit from exposure to applications in quantum information within QuSoft, the Dutch research center for quantum software in Amsterdam. In addition the candidate will be able to collaborate with POEMA partners (Etienne de Klerk) at the University of Tilburg and to follow courses e.g. via the national MasterMath and LNMB programs.

**Planned secondments.** The PhD candidate will have a research stay at CNRS-LAAS in Toulouse, France (Didier Henrion) and with the Numerical Algorithms Group (NAG), Oxford, UK (Mike Dewar).

**Required Skills.** Motivated candidates should hold at the date of recruitment a Master's degree, in Mathematics (preferably), Computer Science or Engineering. The applicant

should have a solid mathematical background, with expertise and taste in several topics including optimization, discrete mathematics, real algebraic geometry and functional analysis. Good programming skills are also a plus. The candidate should have good communication skills and be fluent in English. Interested candidates are kindly asked to send an e-mail with "POEMA candidate" in the title, and the application to [M.Laurent@cw.nl](mailto:M.Laurent@cw.nl) and to also submit their documents at <https://easychair.org/cfp/POEMA-19-22>.