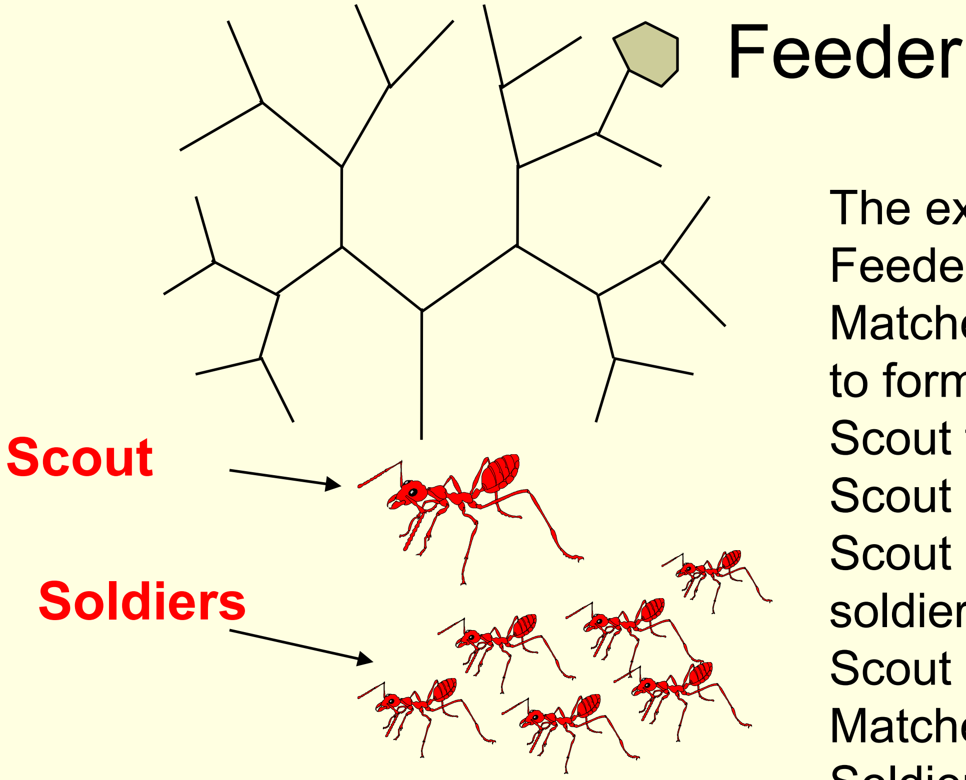
Lecture 8. Kolmogorov complexity and Nature

- In biology, in physics, in science, and in our daily lives, Kolmogorov complexity is everywhere.
- This lecture selects a few beautiful examples.

1. Kolmogorov complexity by ants

Reznikova, Ryabko: When the path to feeder has lower Kolmogorov complexity like ‘LLLL’, ants communicate faster.



The experiment details:
Feeder contains honey.
Matches float on water
to form the tree maze.
Scout first finds honey.
Scout returns.
Scout communicates with
soldier ants, time recorded.
Scout is then removed.
Matches replaced.
Soldier ants go for honey

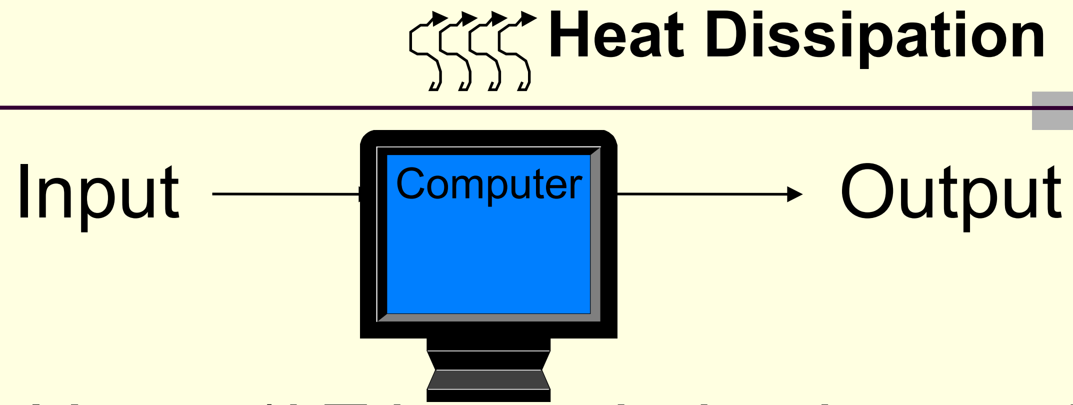
Information compression by ants (using tactile code)

No.	Path	Mean time	Deviation	# Tests
1	LLL	72	8	18
2	RRR	75	5	15
3	LLLLLL	84	6	9
4	RRRRR	78	8	10
5	LLLLL	90	9	8
6	RRRRRR	88	9	5
7	LRLRLR	130	11	4
8	RLRLRL	135	9	8
9	LLR	69	4	12
10	LRLL	100	11	10
11	RLLLR	120	9	6
12	RRLRL	150	16	8
13	RLRRRL	180	20	6
14	RRLRRR	220	15	7
15	LRLLRL	200	18	5

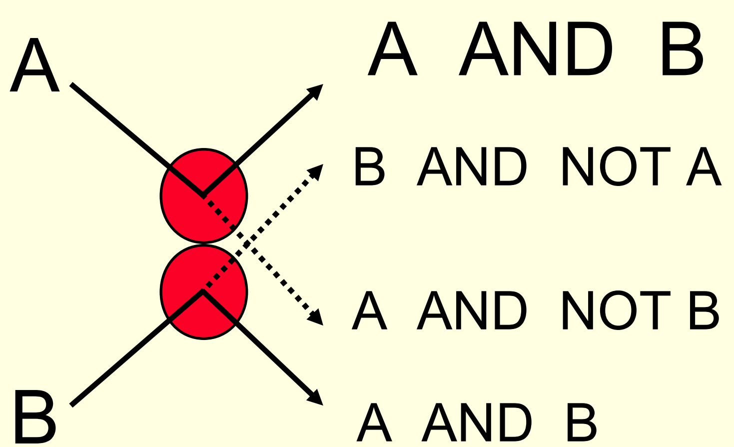
2. Saving 2nd law of thermodynamics

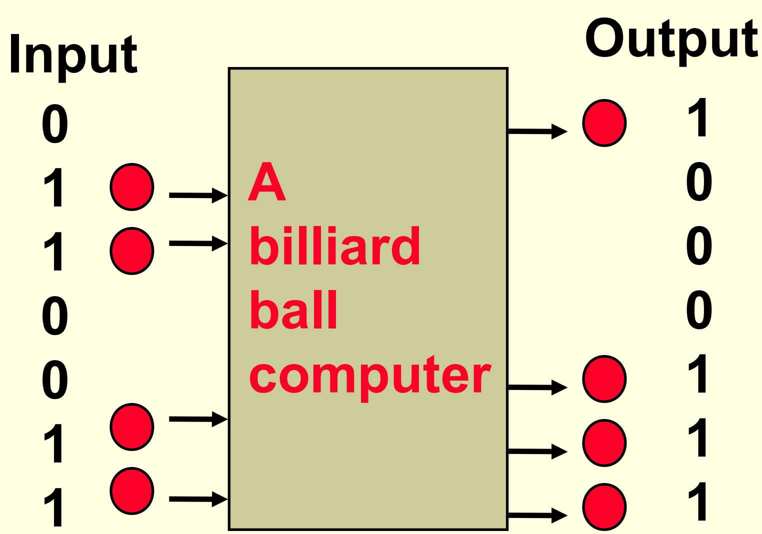
- Two fundamental laws of thermodynamics:
  - 1st law: The total energy of an isolated system is invariant over time
  - 2nd law: No process is possible that has its only result the transformation of heat into work
- But the 2nd law has suffered serious problems for over 100 years until Kolmogorov complexity is used recently.

Thermodynamics of Computing

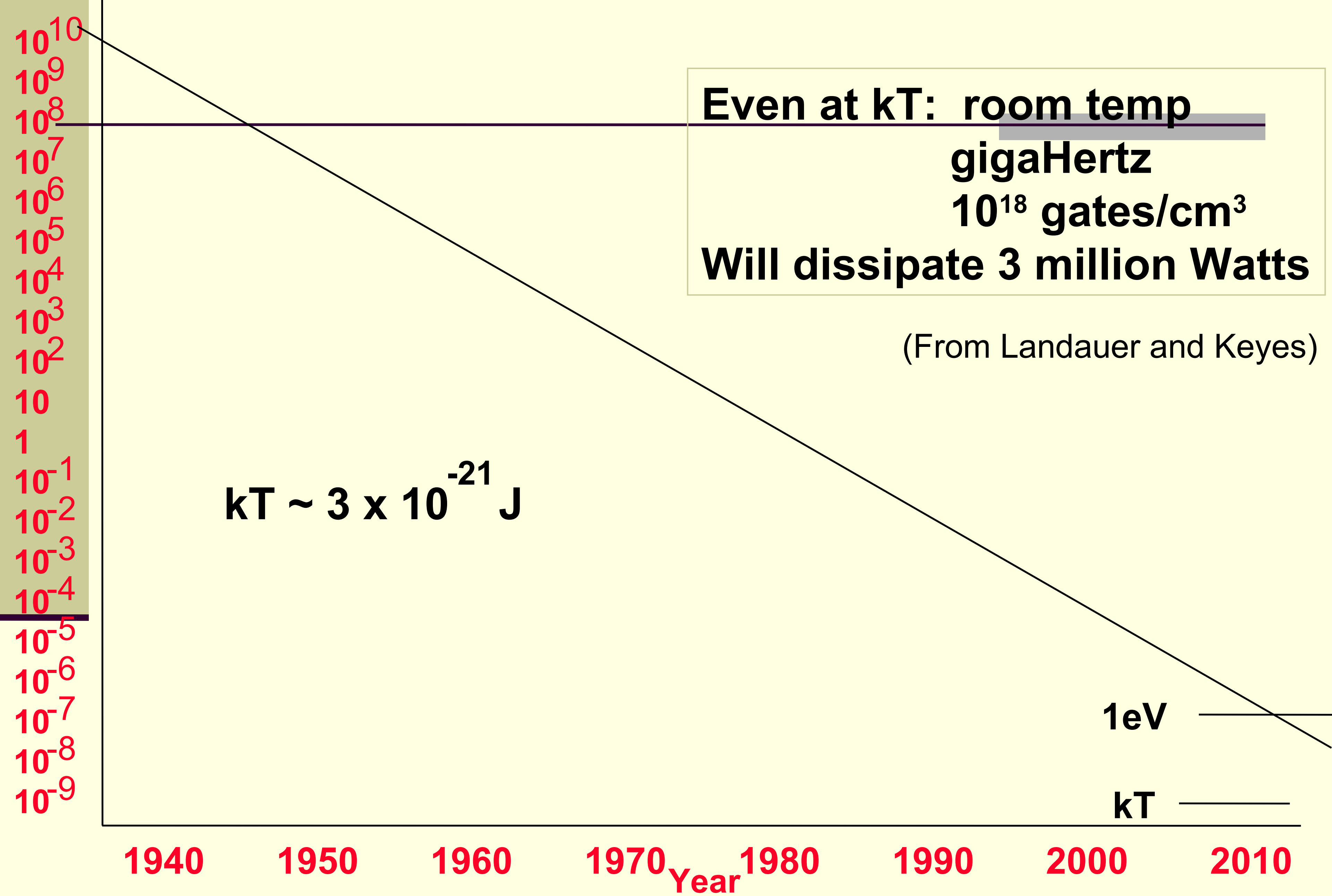


- Physical Law: 1kT is needed to irreversibly process 1 bit (Von Neumann, Landauer)
- Reversible computation is free.





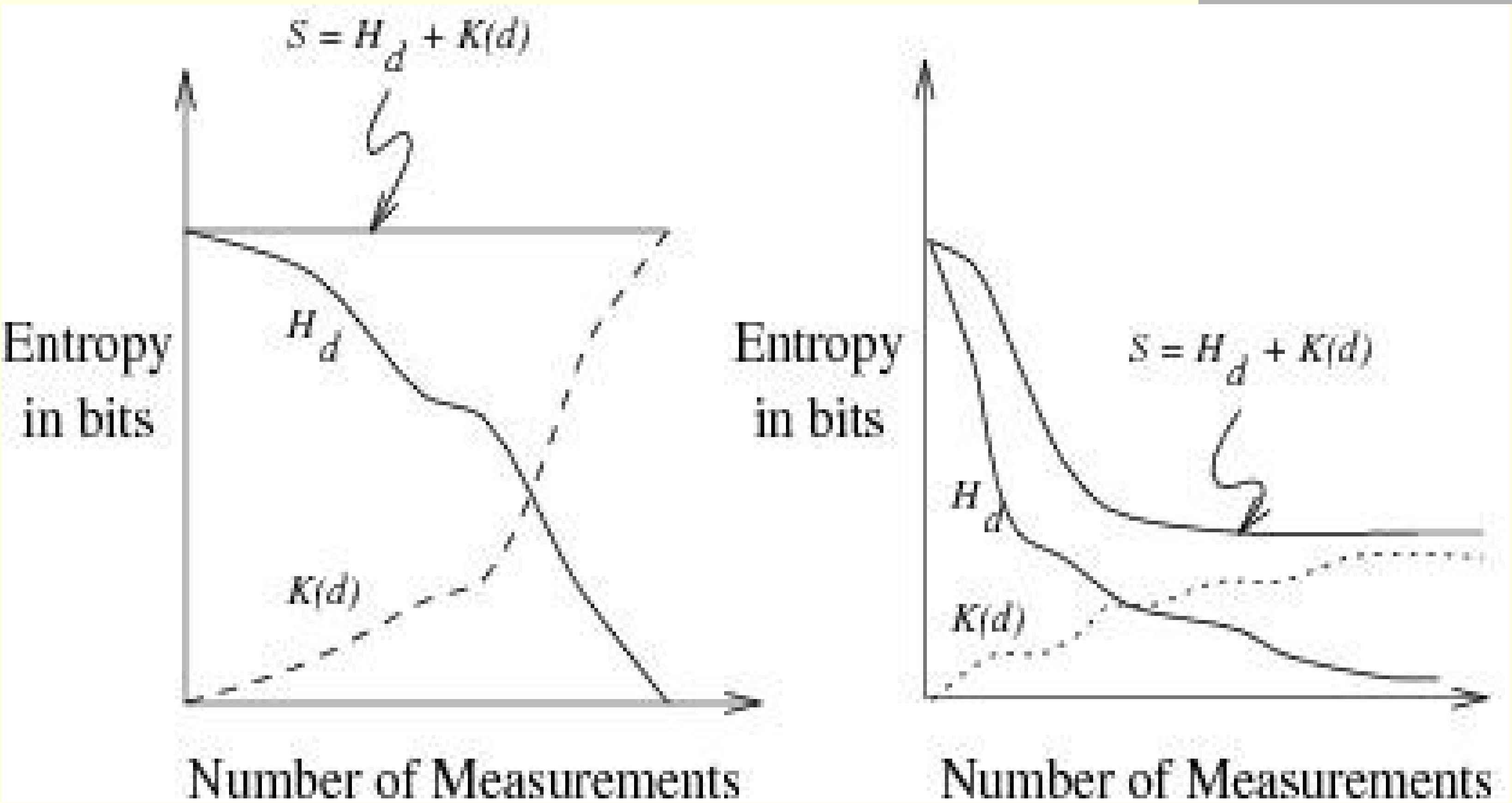
Trend: Energy/Operation



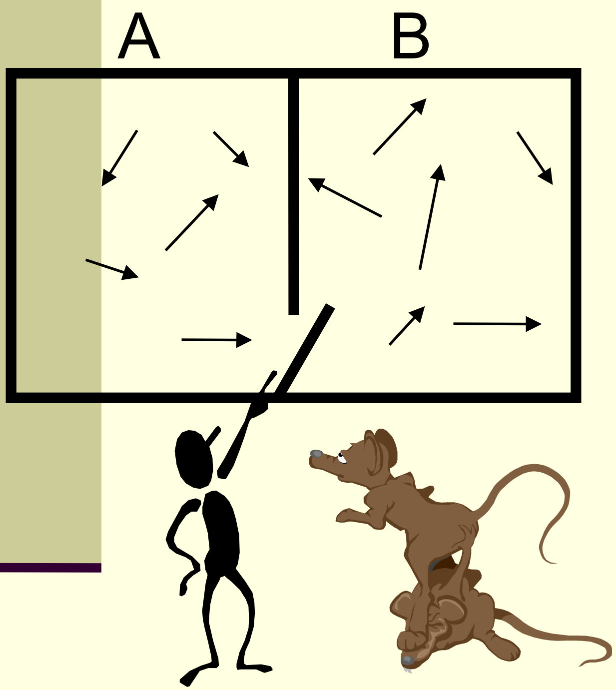
Information is physical

- Ultimate thermodynamics cost of erasing x:
  - "Reversibly compress" x to x* -- the shortest program for x.
  - Then erase x*. Cost C(x) bits.
  - The longer you compute, the less heat you dissipate.
  - More accurately, think reversible computation as a computation that can be carried out backward. Let's look at how we can erase x given (x,x*). Step 1: x*→x, g(x*,x) (these are garbage bits); Step 2: cancel one copy of x; Step 3. x,g(x*,x) → x* (this is reversal of step 1); Step 4: erase x* irreversibly.
- Formalize:
  - Axiom 1. Reversible computation is free
  - Axiom 2. Irreversible computation: 1 unit/bit operation

Zurek's Physical Entropy

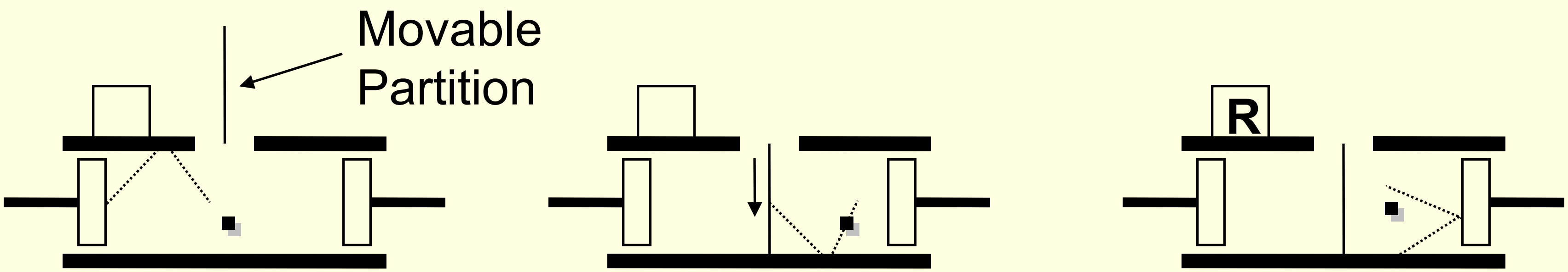


Maxwell's Demon



J.C. Maxwell, 1871, Theory of Heat:
If we conceive a being whose faculties are so sharpened that he can follow every molecule … opens and closes this hole so as to allow only swifter molecule to pass from A to B, slower ones from B to A. He will thus without expenditure of work, raise temperature of B and lower that of A, in contradiction to the 2nd law of thermodynamics.

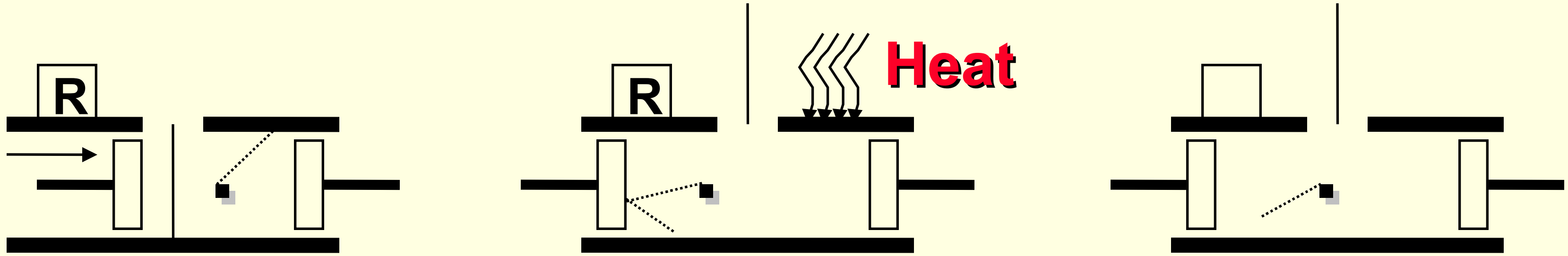
Solution to Maxwell’s Demon (Bennett)



Initial state

Molecule trapped

Record: right side



Left piston pushed in for free.

Separator lifted, molecule pushes left piston to left using heat from environment.

Memory is erased, back to initial state.

Szilard Engine and its information-theoretic explanation.

3. Entropy & Algorithmic Physics

- On his grave stone, engraved is Boltzmann's famous entropy formula: S=*k*logN, where $k=1.38 \times 10^{-23}$ joules/Kelvin is the Boltzmann constant.
- Here N is the number of possible states in the system. It is sometimes more convenient to express such entropy in terms of Kolmogorov complexity. Let us consider some examples.

Determinism versus Probability

- For the great probabilist P.S. Laplace, the notion of probabity just serves to account for our ignorance concerning the multitude of deterministic causes.
- Einstein did not believe in random variables (or quantum mechanics) `I do not believe that the Lord plays dice'
- G. 't Hooft believes that essentially nature is deterministic

Entropy

- In Physics, one often uses `high or low entropy' to state that a system is in `disorderly or orderly state'.
- This is incorrect terminology. If the system is deterministic, the entropy $\sum p \log 1/p$ is always 0, since all probabilities are 0 except of that of the state concerned which is 1.
- Even if the system were a random variable, high entropy simply means a lot of possible outcomes, and low entropy means less possible outcomes, but both ordered and disordered outcomes are possible (although more for high entropy).

Laws of Nature

- The world is (can be encoded as) a sequence of bits. Every random sequence must have subsequences that are regular (for binary random sequences of length n we must have a run of 1's or 0's of length log n)
- Even if the Universe is random in the large, there must be parts that are regular, i.e. Satisfy definite simple laws. We may live in such a regular part; indeed, the antropomorphic argument says we do.
- Now we need to formalize these ideas

Loschmidt Paradox

(born, 1821-1895, first estimation of Avogadro number, Boltzmann’s colleague)

- How does entropy increase in a deterministic system?
- That is: if you reverse time (time symmetry holds for almost all known low level fundamental physics process), then entropy decreases?

Example 1. Superconductivity

- In high temperature superconductivity research, a material like CuO_2 loses magnetic moment below a critical temperature. In such a state, the nuclear spins all line up as below:

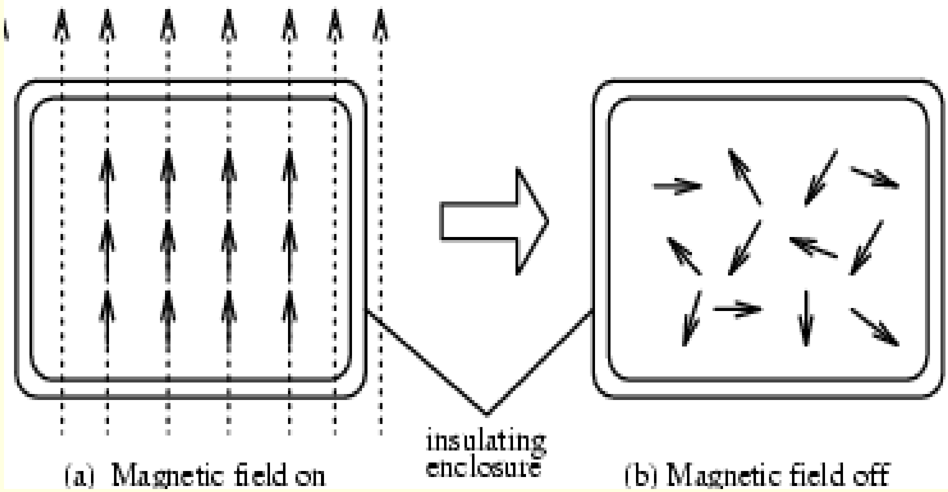
  ↑↓↑↓↑↓↑↓↑↓↑…
  ↓↑↓↑↓↑↓↑↓↑↓…
  ↑↓↑↓↑↓↑↓↑↓↑…
  ↓↑↓↑↓↑↓↑↓↑↓…

- This low entropy is most naturally expressed by Kolmogorov complexity with a short program:

  repeat forever print ↑; **print** ↓;

Example 2. Cooling down

- Adiabatic demagnetization is an important technique that has been used to achieve record low temperatures – near zero Kelvin.
- Chrome-alum salt (whose molecules may be considered as tiny magnets) is placed in a thermally insulating (adiabatic) enclosure.
- A strong magnetic field is applied by an external magnet so that the tiny atomic magnet (spins) line up, forming low Kolmogorov complexity state.
- Then the magnet is removed so the spins becomes chaotic again --- entropy (Kolmoogorov complexity) increasing implies absorbing energy (heat), hence lowering the temperature.
- This process is repeated …



Project, research topic

- Kolmogorov complexity interpretation of chaos—see Vitanyi 2007
- Experimental project: verify the ant experiment! Or with bees? See Resznikova’s CUP book 2007