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It is interesting to consider the excursions of mathematicians and scientists into prose and poetry, and, conversely and less known, the explorations of poets and novelists into mathematics.

An example of the first is the Dutch mathematician/logician Luitzen E. J. Brouwer's excursion into literature and environmentalism [1], an early appeal to save the earth's natural environment from human pollution. In particular, he wants to abolish the technology that enables man's supremacy over nature and the physics and mathematics that make this possible. All that he would save from this is pure ("intuitionistic") mathematics, which by its nature is inapplicable for evil purposes, and which is the ultimate creation of the noble mind.

In another direction, the great Russian mathematician Andrei Nikolaevich Kolmogorov was particularly interested in the form and structure of the poetry of Pushkin [3]. He also remarks [4], "What real meaning is there, for example, in asking how much information is contained in 'War and Peace'? Is it reasonable to include this novel in the set of 'possible novels,' or even to postulate some probability distribution for this set? Or, on the other hand, must we assume that the individual scenes in this book form a random sequence with 'stochastic relations' that damp out quite rapidly over a distance of several pages?"

Tolstoy's answer to the last question is decidedly "no." There is a ubiquitous general theme in *War and Peace*, namely, the idea that single individuals cannot influence in any direction the course of history (contrary to what is assumed in most history writing), but that the course of history is determined by the confluence of myriad infinitesimally small individual human acts of free will, much as a flock of birds wheels about in synchrony without any apparent governor. He regards individual humans as interchangeable atoms of an ideal gas that in combination determine effects on macroscopic scales such as heat and pressure—as in the nineteenth-century statistical physics of H. von Helmholtz. He invokes this idea to justify his egalitarian convictions. Helmholtz is also the author of the unrelated witticism, so true and so unknown to politicians and administrators of science: "Whoever in the pursuit of science, seeks after immediate practical utility may rest assured that he seeks in vain" [2].

It is unknown whether and perhaps unlikely that the great Russian novelist Count Leo Nikolayevich Tolstoy was aware of Helmholtz's work (or directly related work); however, he may well have known Laplace's [5] *Essay* describing the world as a mechanistic ensemble of moving and colliding particles that by their combined microscopic actions produce macroscopic effects. This is likely, given the immense influence of Laplace's work, and given Tolstoy's interest in justice and believable testimony in his role as country noble passing judgment on his people. The *Essay* in fact treats in great detail matters of believable testimony and probability of proper justice—live issues at that time.

Tolstoy's concern is unknowability and uncertainty: he is not really seeking a usable model so much as a reductio ad absurdum to show the futility of the quest for explanations of wars' outcomes. His conclusion is not that the future is in the laps of the gods, but rather that it is decided deterministically and precisely, but is practically (and possibly in principle) unknowable by humans. One recalls that Laplace in the *Essay* says that a demon knowing the positions, masses, and velocity vectors of all particles can perfectly predict the future and reconstruct the past, but the imperfect human mind cannot access all information about the present at once, and so is reduced to ignorance or at best to probabilistic reasoning.

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The author of *War and Peace* had an intense interest in mathematical approaches to the sciences, as appears from his proposals to found sociology, history, and the science of war as a mathematical discipline, just as mathematician John von Neumann later proposed to found the science of economics as a mathematical discipline in [11].

Tolstoy's views on the matter are found in *War and Peace*, by many regarded as one of the greatest novels in any language. They are set forth passionately in long interludes scattered through the latter parts of the novel.

Recall that the book follows the doings and adventures of a group of aristocratic people, and the descriptions of great battles, at the time of Napoleon's invasion in the bleak reaches of great Russia. Closer inspection reveals that one of the main themes of the tale is the insignificance and expendability of the particular heroes—such as Napoleon—in the sweep of history: the events would have unfolded in the same way irrespective of the so-called main figures.

War and Peace was first published from 1865 to 1869 in *Russkii Vestnik* (Russian Messenger), the literary journal of editor Mikhail Katkov (which distinguishes it from two other Russian magazines with the same name in the nineteenth century). Thus developments in the mathematization of the behavioral sciences after those dates could not have contributed to the ideas in the novel.

Below I quote liberally from *War and Peace* to present Tolstoy's exact phrasing that shows his point of view. I will use Rosemary Edmonds's 1957 translation into English published by Penguin Classics [8] (part I, 1972 printing; part II, revised 1978 printing). References to page numbers will be made in the form [WP, xx].

A New Approach to History

Tolstoy disagrees with the view of history that ascribes the evolution of events to individuals:

> One might have supposed that the historians, who ascribe the actions of the masses to the will of one man, would have found it impossible to fit the flight of Napoleon's armies into their theory, considering that during this period of the campaign [in Russia] the French did all they could to bring about their own ruin, and that not a single movement of that rabble of men . . . betrayed a hint of rhyme or reason. But no! Mountains of volumes have been written by historians . . . [with] accounts of Napoleon's masterly arrangements and deeply considered plans . . . [WP, 1266]

Not only can individuals not be the main governors of the making of history, but "It is beyond the power of the human intellect to encompass *all* the causes of a phenomenon." . . . "the human intellect . . . snatches at the first comprehensible approximation to a cause and says: 'There is the cause'." Tolstoy goes on [WP, 1168] to explain that "in historical events (where the actions of men form the subject of observation) the primeval conception of a cause was the will of the gods, succeeded later on by the will of those who stand on the historical foreground—the heroes of history."

On page [WP, 1342] Tolstoy continues to inveigh against common views of history:

> Why did things happen thus, and not otherwise? Because they did so happen. '*Chance* created the situation; *genius* made use of it,' says history. But what is *chance*? What is *genius*? The words *chance* and *genius* do not denote anything that actually exists, and therefore they cannot be defined. These two words merely indicate a certain degree of comprehension of the phenomena. I do not know why a certain event occurs; I suppose that I cannot know-therefore I do not try to know, and I talk about *chance*. I see a force producing effects beyond the scope of ordinary human agencies; I do not understand why this occurs, and I cry *genius*.

Rather we should try to treat history in the spirit of the so successful natural sciences. The "unreasonable effectiveness of mathematics in natural science" as phrased by E. P. Wigner [12], must be extended *avant la lettre* to sociology and political history [WP, 977]:

> To elicit the laws of history we must leave aside kings, ministers, and generals, and select for study the homogeneous, infinitesimal elements which influence the masses. No one can say how far it is possible for a man to advance in this way to an understanding of the laws of history; but it is obvious that this is the only path to that end, and that the human intellect has not, so far, applied in this direction one-millionth of the energy which historians have devoted to describing the deeds of various kings, generals and ministers, and propounding reflections of their own concerning those deeds.

How then is this proper view of history obtained? Tolstoy refers to mechanics, where continuity of motion can be captured by reasoning dividing the continuous into units. He is aware of the danger of going astray [WP, 974]: Recalling the fallacy of Achilles and the tortoise, he says,

> The absurdity of the finding (that Achilles can never overtake the tortoise) follows from arbitrarily separating the motion into separate units, whereas the motion of Achilles and the tortoise was continuous.
>
> By adopting smaller and smaller units of motion we only approximate the solution of the problem but never reach it. It is only by admitting infinitesimal quantities and their

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progression up to a tenth, and taking the sum of that geometrical progression, that we arrive at the solution of the problem.

This leads to the heart of the matter: Tolstoy's proposal for a differential and integral analysis of history [WP, 974–975]:

> A new branch of mathematics, having attained the art of reckoning with infinitesimals, can now yield solutions to other more complex problems of motion which before seemed insoluble. This new branch of mathematics, which was unknown to the ancients, by admitting the conception, when dealing with problems of motion, of the infinitely small and thus conforming to the chief condition of motion (absolute continuity), corrects the inevitable error which human intellect cannot but make if it considers separate units of motion instead of continuous motion. In the investigation of the laws of historical movement precisely the same principle operates.
>
> The march of humanity, springing as it does from an infinite multitude of individual wills, is continuous. The discovery of the laws of this continuous movement is the aim of history. But to arrive at these laws of continuous motion resulting from the sum of all those human volitions, human reason postulates arbitrarily, separated units. The first proceeding of a historian is to select at random a series of successive events and examine them apart from others, though there is and can be no *beginning* to any event, for an event flows without break in continuity from another. The second method is to study the actions of some one man—a king or a commander—as though their actions represented the sum of many individual wills; whereas the sum of the individual wills never finds expression in the activity of a single historical personage.
>
> . . . Only by assuming an infinitesimally small unit for observation—a differential of history (that is, the common tendencies of men)—and arriving at the art of integration (finding the sum of the infinitesimals) can we hope to discover the laws of history.

Mathematics of War

The causality involved in war defies simple analysis, Tolstoy says, but is reached by the integration of the infinitesimal individual causes [WP, 1184]:

> An infinite number of freely acting forces (and nowhere is a man freer than during a life and death struggle) influence the course taken by a battle, and that course can never be known beforehand and never coincides with the direction it would have taken under the impulsion of any single force.

We cannot quite follow Tolstoy here: the expectation of an ensemble might happen to coincide with one of its elements. But worse is to come.

In [WP, 1223—1224] Tolstoy outlines the mathematics of war and goes into an explicit calculation that is patently false:

> Military science says, the greater the numbers [of an army] the greater the strength. . . . For military science to make this assertion is like defining energy in mechanics by reference to the mass only. It is like saying that the momenta of moving bodies will be equal or unequal according to the equality or inequality of their masses. But momentum (or 'quantity of motion') is the product of mass and velocity. So in warfare the strength of an army is the product of its mass and of something else, some unknown factor x.

After rejecting some common explanations of what this x may be, in particular the interpretation of x as the amount of genius of the commanding general, he goes on to say that [WP, 1224]

> We must accept the unknown and see it for what it is: the more or less active desire to fight and face danger. Only then, expressing the known historical facts by means of equations, shall we be able to compare the relative values of the unknown factor; only then may we hope to arrive at the unknown itself.
>
> If ten men, batallions or divisions, fighting fifteen men, batallions or divisions, beat the fifteen—that is, kill or capture them all while losing four themselves, the loss will have been four on one side and fifteen on the other. Therefore, the four are equal to the fifteen, and we may write $4x = 15y$. In other words, x is to y as 15 is to 4. Though this equation does not yet give us the absolute value of the unknown factor, it does give us a ratio between two unknowns. And by putting a whole variety of historical data (battles, campaigns, periods of warfare, and so on) into the form of such equations, a series of figures will be obtained which must involve the laws inherent in equations and will in time reveal them.

This argument of Tolstoy is remarkable. He compares the losses of the conquering army with the total of the vanquished army—perhaps on the grounds that the vanquished army is totally lost. Testing the idea by inserting more extreme figures, such as that an army of 1,000,000 men beats a small army of 10 men, while the conquering army loses one man, we obtain the equation $x = 10y$. With Tolstoy's interpretation, this would seem to mean that the fighting spirit of the million-man army exceeded the fighting spirit of the minuscule ten-man army tenfold. Basing our computation on the ratio of the losses of the conquering army (irrespective of the total size of that army) to the total of the beaten army (however small) does not work at all, as we have seen by considering extreme numbers.

A New Approach to History, Revisited

Part Two of the Epilogue of *War and Peace* [WP, 1400—1444] again gives Tolstoy's view of the force that moves nations, a sociology of history. The question he analyzes is again that of the power of leaders versus the will of the masses. The basic concepts are free will, consciousness, cause, and necessity. Time is an essential:

> A contemporary event seems to us indubitably the doing of all men we know of concerned in it; but in the case of a more remote event we have had time to observe its inevitable consequences, which prevents our conceiving of anything else as possible. And the farther back we go in our investigations of events the less arbitrary do they appear. [WP, 1433]
>
> . . . the force of freewill constitutes the subject-matter of history. . . . what is known to us we call laws of necessity; what is unknown we call freewill. . . . [We recognize]

man's freewill as a force capable of influencing historical events, that is, as not subject to laws [WP, 1440].

Only by reducing this element of freewill to the infinitesimal, that is, by regarding it as an infinitely small quantity, can we convince ourselves of the absolute inaccessibility of causes, and then instead of seeking causes history will adopt for its task the investigation of historical laws. ... Reaching the infinitesimal or infinitely small, mathematics—the most exact of the sciences—leaves off dividing and sets out upon the new process of integrating the infinitesimal unknown. Abandoning the concept of causation, mathematics looks for laws, i.e., the properties common to all infinitely small unknown elements.

... if the subject of history is to be the study of the movement of nations and humanity, and not the description of episodes in the life of individuals, it too is bound to lay aside the notion of cause and seek the laws common to all the equal and indissolubly interconnected infinitesimal elements of freewill. [WP, 1441–1442]

Plausibility of the Ideas

The attempted mathematization of history rests on the superficial application of the calculus of infinitesimals. The author uses only the basic ideas—in a rather sketchy form. Tolstoy tentatively describes a historical science and a behavioral science that are mathematized in the way the physical sciences have been mathematized since Newton's time, complete with the use of infinitesimals, differentiation, and integration, in short, a calculus of behavioral sciences and history. There is no suggestion of anything like theorems or proofs. This theory is not developed further by Tolstoy, nor do I see that it directly influenced later mathematics of the social sciences. Though his outline seems sensible, a serious mathematical development of the concepts must have appeared daunting or infeasible. Any useful theory along his lines—let alone application—lies still in the future.

A question that arises is the following. Tolstoy's central tenet states that outcomes are the product of the combined working of the free will of the people, rather than of decisions by a leader. Obviously, Tolstoy did not know such "heroes" of the twentieth century as Lenin, Stalin, and Hitler, but he certainly knew Alexander the Great and Genghis Khan. One would think that his analysis ought to give at least some special role to such leaders. In war, one would think that his x and y might involve the quality of commanders (as the quality of performance of a symphony orchestra may have special dependence on the quality of the conductor). Even by raising the possibility of analyzing rationally the relation between leaders and followers, between intention and collective action, Tolstoy anticipates modern treatments [6].

If we interpret Tolstoy's model as predicting outcomes to be resultants of myriad individual pushes, we surely must find this from our perspective to be an unrealistically linear model—vector addition—where various factors must interact more intricately. It is ironic that his effort to insist on a greater complication of historical processes still left him with an overly simple picture. Today a diversity of mathematical models is in use (some of them surely also too simple and too linear), and perhaps we may regard them as catching up to the dream Tolstoy had more than a century ago.

Tolstoy's Motivation

In his old age, Tolstoy freed his serfs, and renounced his estate, his title, more or less his wife, and his copyrights. The same singleness of purpose and strength of conviction were expressed earlier in his unique attempt to apply scientific notions in novels. He dared to apply mathematics to history and the science of war, as a lone pioneer.

One may view this as infatuation with scientific and mathematical ideas on the part of an interested layman. This is unusual enough for a literary man. But it seems that he had eccentric ideas even in areas where he was an expert, such as literature and art. Certain of his opinions as expressed in his writings are highly idiosyncratic.

In [9], Tolstoy discusses the notion of "art." In his opinion good art should "infect" the receiver with feeling, more especially of universal brotherhood, but bad art does not. These feelings are only transmitted by Christian art (but not by art created by artistic elites because they have lost the core of Christianity). For example, ancient Greek art is not good since it does not express the values of Christianity.

Among western artists, he condemns, among others,

- in literature: Shakespeare, Dante, Milton, and much of Pushkin;
- in painting: Raphael, Michelangelo (including the "absurd" Last Judgment);
- and in music: almost all of Bach, Beethoven, and Wagner.

He specifically mentions Beethoven's 9th Symphony: it cannot "infect" the audience as it suggests a feeling of unity and cannot therefore be good art.

He sees the need for special treatment of Shakespeare. His pamphlet [10] attacking Shakespeare surprised many English readers. He says there that Shakespeare has always aroused in him "an irresistible repulsion and tedium." In the course of his life he has read and re-read many of Shakespeare's works in Russian, English, and German. At seventy-five, he re-read the entire works of Shakespeare including the historical plays. The conclusion was that Shakespeare was no genius, not even an average author. To illustrate his disgust he selects and analyzes a particular play, *King Lear*, using his theory of art [9]. In a rebuttal written about 40 years later, George Orwell [7] calls Tolstoy's criticisms, quotations, and generalizations malicious. He ends by stating that Shakespeare's plays are still popular after 200 years (actually by now about 400 years), whereas Tolstoy would be forgotten if he had not been the author of two of the greatest works of the nineteenth century, *War and Peace* and *Anna Karenina*.

These caprices tell us that Tolstoy was not afraid but was more than willing to voice eccentric opinions. This is of a piece with his venturing to use science and mathematics in support of his speculations in *War and Peace*.

Conclusion

It is rare indeed for a great author to incorporate extensive discussions about mathematical foundations of social sciences in a major novel. Much more common are ventures

by scientists in a literary direction. Tolstoy, uniquely, set forth a program to mathematize history, sociology, and the science of war in line with the rational inclination of the nineteenth century.

He rejects explaining historical events in terms of the will of leaders. He proposes to make individual acts of free will of ordinary people infinitesimal, and to express historical events by integration of those individual efforts. He dreamt of this program, as evidenced by the remarks I have quoted above, but did not carry it out. It has not been carried out by others either. Some of modern mathematics of the social sciences uses continuous models. But we do not see the passage from discrete to continuous in the manner of nineteenth-century mechanics.

When Tolstoy tries to model a simple conflict, as cited previously, he does it badly. Even if he had avoided the blunder, his proposed model would have been limited in that it used only very few parameters, contrary to the spirit of his general theoretical remarks.

Modern hindsight certainly does not make Tolstoy's theories look prophetic. As a challenge they are indeed ahead of their time. At least the challenge may have influenced theoretical development of the twentieth century, and the future.

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