

MDL exercises, seventh handout
(due April 11th)

1. The prequential plug-in code based on the ML estimator is a universal code. Below we only consider one-dimensional models such that the data are i.i.d. according to all distributions in the model. For such models, the ML estimator is usually defined after 1 observation. Thus the prequential codelength for the second outcome given the first is defined, but the prequential outcome for the first outcome is not defined. Below you may ignore the contribution of that outcome to any codelengths you compute. Just pretend that the contribution of the first outcome is zero. Thus, the regret of an i.i.d. Poisson sequence x^n becomes the prequential codelength of outcomes x_2, \dots, x_n minus the codelength for outcomes x_2, \dots, x_n based on the ML estimator for all outcomes.

Question: consider the prequential ML code for the Poisson model. Show that the regret can be infinite.

2. a) Show that for the Poisson model, when $n = 1$ (1 outcome), the minimax regret is infinite (Hint: use Stirling's approximation!) and hence the NML distribution is undefined. For bonus points, you may also show that the NML is also undefined for data of arbitrary fixed length (all $n > 1$).
- b) Explain why, for the Poisson model restricted to parameters $\mu \in [1, 100]$ and data of length $n = 1$, the minimax regret is finite and hence NML is defined. For bonus points, you may once again show that the same holds for any fixed $n > 1$.
- c) Show that for the Poisson model restricted to parameters $\mu \in [1, 100]$ for all large enough n , there is a sequence of data for which the prequential plug-in code has regret that is *smaller* than the regret of the NML code by a term of order $(1/2) \log n$. Does this show that prequential plug-in code is really preferable over the NML code?