1. Suppose Alice and Bob share an EPR-pair $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

(a) Let $U$ be a unitary with real entries. Show that the following two states are the same:
   (1) the state obtained if Alice applies $U$ to her qubit of the EPR-pair;
   (2) the state obtained if Bob applies the transpose $U^T$ to his qubit of the EPR-pair.

(b) What state do you get if both Alice and Bob apply a Hadamard transform to their qubit of the EPR-pair? Hint: you could write this out, but you can also get the answer almost immediately from part (a) and the fact that $H^T = H^{-1}$.

2. Give a classical strategy using shared randomness for the CHSH game, such that Alice and Bob win the game with probability at least $3/4$ for every possible input $x, y$ (note the order of quantification: the same strategy has to work for every $x, y$). Hint: For every fixed input $x, y$, there is a classical strategy that gives a wrong output only on that input, and that gives a correct output on all other possible inputs. Use the shared randomness to randomly choose one of those deterministic strategies.

3. Consider three space-like separated players: Alice, Bob, and Charlie. Alice receives input bit $x$, Bob receives input bit $y$, and Charlie receives input bit $z$. The input satisfies the promise that $x \oplus y \oplus z = 0$. The goal of the players is to output bits $a, b, c$, respectively, such that $a \oplus b \oplus c = \text{OR}(x, y, z)$. In other words, the outputs should sum to $0$ (mod 2) if $x = y = z = 0$, and should sum to $1$ (mod 2) if $x + y + z = 2$.

(a) Show that every classical deterministic strategy will fail on at least one of the 4 allowed inputs.

(b) Show that every classical randomized strategy has success probability at most $3/4$ under the uniform distribution on the four allowed inputs $xyz$.

(c) Suppose the players share the following entangled 3-qubit state:

$$\frac{1}{2}(|000\rangle - |011\rangle - |101\rangle - |110\rangle).$$

Suppose each player does the following: if his/her input bit is 1, apply $H$ to his/her qubit, otherwise do nothing. Describe the resulting 3-qubit superposition.

(d) Using (c), give a quantum strategy that wins the above game with probability 1 on every possible input.