Visualization of Global Flow Structures Using Multiple Levels of Topology

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Abstract

The technique for visualizing topological information in fluid flows is well known. However, when the technique is used in complex and information rich data sets, the result will be a cluttered image which is difficult to interpret. This paper presents a technique for the visualization of multi-level topology in flow data sets. It provides the user with a mechanism to visualize the topology without excessive cluttering while maintaining the global structure of the flow.

Keywords: flow visualization, multi-level visualization techniques, flow topology, turbulent flow.

1 Introduction

The importance of data visualization is clearly recognized in large scale scientific computing. However, the demands imposed by modern computational fluid dynamics (CFD) simulations severely test the limits of today's visualization techniques. This trend will continue as solutions to more complex problems are desired.

An important property of a flow field is its topology [1]. Visualization of the topology was introduced into flow visualization by Helman and Hesselink [2]. They presented a technique that extracts and visualizes topological information from numerical flow simulations. Topology visualization combines simplicity of schematic depictions with the quantitative accuracy of curves computed directly from the data. The technique presented by Helman and Hesselink works well when applied to simple flow fields, but when applied to complex fields, such as turbulent flows, a problem arises. Turbulent flows are characterized by many small disturbances, resulting in a very large set of critical

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points. The visualization of the complete topology will be a cluttered image which is difficult to interpret. For example, consider figure 1. The data is a 2D slice of a 3D turbulent flow around a square cylinder. Spot noise is used to present the global nature of the flow. A set of 322 critical points has been found in the data. Colored icons are used to display the set of critical points: a yellow spiral icon denotes a focus, a blue cross denotes a saddle point, and cyan/magenta disks denote repelling/attracting nodes. To prevent additional cluttering, streamlines linking critical points have been omitted. Note that most critical points are clustered in regions around the square cylinder.

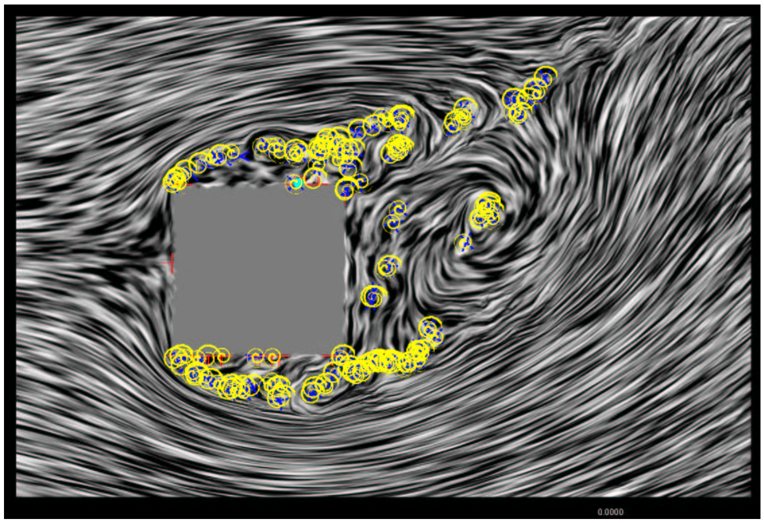

Fig. 1. Topological information of turbulent flow around a square cylinder. Colored icons are used to represent critical points. Spot noise is used to present the global nature of the flow.

In this paper we present a technique for the visualization of multi-level topology in flow data sets. The multi-level topology technique provides a mechanism to select the set of critical points which define the global flow structure. By only displaying the selected set of critical points, a simplified depiction of the topology can be given, while maintaining the global nature of the flow field.

The format of this paper is: In the next section we review previous work on vector field topology. In section 3 discusses the multi-level topology technique. Finally, in section 4.1 we show how the technique has been used in two applications: to explore a turbulent flow field and to explore wind fields.

2 Previous work

Vector field topology was introduced by Helman and Hesselink, [2]. It presents essential information by partitioning the flow field in regions using critical points which are linked by streamlines. Critical points are points in the flow where the velocity magnitude is equal to zero. Each critical point is classified based on the behavior of the flow field in the neighborhood of the point. For this classification the velocity gradient tensor is used. The velocity gradient tensor – or Jacobian – is defined as

$$\mathbf{J} = \nabla \vec{u} = \begin{pmatrix} u_x & u_y \\ v_x & v_y \end{pmatrix} \tag{1}$$

in which subscripts denote partial derivatives. The classification is based on the the two complex eigenvalues ($R1 + i\ I1,\ R2 + i\ I2$). Assuming that the the critical point is hyperbolic, i.e. the real part of the eigenvalues is non zero, five different cases are distinguished (see figure 2) :

(1) *Saddle point*, the imaginary parts are zero and the real parts have opposite signs; i.e $R1 * R2 < 0$ and $I1\ ,\ I2 = 0$.
(2) *Repelling node*, imaginary parts are zero and the real parts are both positive; i.e $R1\ ,\ R2 > 0$ and $I1,\ I2 = 0$.
(3) *Attracting node*, imaginary parts are zero and the real parts are both negative; i.e $R1\ ,\ R2 < 0$ and $I1,\ I2 = 0$.
(4) *Repelling focus*, imaginary parts are non zero and the real parts are positive; i.e $R1\ ,\ R2 > 0$ and $I1,\ I2 \neq 0$.
(5) *Attracting focus*, imaginary parts are non zero and the real parts are negative; i.e $R1\ ,\ R2 < 0$ and $I1,\ I2 \neq 0$.

When the real part of the Eigen values is zero the type of the flow is determined by higher order terms of the approximation of the flow in the neighborhood of the critical point.

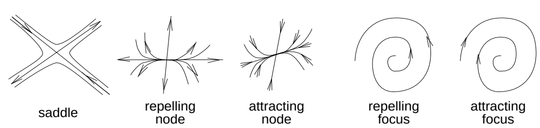

Fig. 2. Five different types of critical points.

Streamlines traced in the direction of the eigenvectors of the velocity gradient tensor will divide the flow field in distinct regions.

Implementation aspects of this technique can be found in [3].

3 Multi-level flow topology

Phenomena in turbulent flow fields are characterized by flow patterns of widely varying spatial scales. In terms of topological information, this means that flow patterns with small spatial scales result in a large set of critical points. However, the global structure of the flow field can be described by a subset of all critical points. The governing idea of the multi-level flow topology method is that the displayed number of critical points should be limited to only those which characterize flow patterns of a certain level of scale. The critical points that determine flow patterns at a smaller spatial scale should not be displayed.

In order to realize this idea, two distinct type of methods can be employed: implicit and explicit methods (see figure 3). Implicit methods are those that filter the input data set to obtain a derived data set to which the original topology algorithm applied. Explicit methods are those which first compute the set of critical points from the input data set and then use a filter to prune this set.

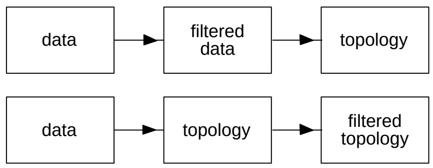

Fig. 3. Implicit (top) vs. explicit (bottom) methods of multi-level topology.

For explicit filtering methods we distinguish between two cases: *local* and *non-local* methods. Local filters are those which operate on the critical points only, and do not use additional information extracted from the flow field. In contrast, non-local filters make use of additional information extracted from the flow field.

The filter described in this paper is a local filter. An example of a non-local filter can be found in [4]. This filter is based on the area of the flow region in the neighborhood of a critical point. Flow regions with small areas are contracted into a single critical point. The topology can be simplified by iteratively contracting regions until a specified area threshold is reached.

3.1 Implicit methods

In general, filters are used to enhance/suppress patterns in the data. For example, a low pass filter can be used to suppress high-frequency patterns; i.e. those flow patterns at small spatial scales. The idea is that by filtering the data and then doing the critical point analysis, the critical points caused by small disturbances in the flow will be filtered out. Nielson et al. obtained similar results with a method where wavelets were used to approximate the data [5].

For the images in this section, a simple box filter has been used: each datapoint is replaced by an average over a small region in the original data. The motivation is that the box filter is a simple low-pass filter which will average out small scale patterns, while keeping large scale patterns intact.

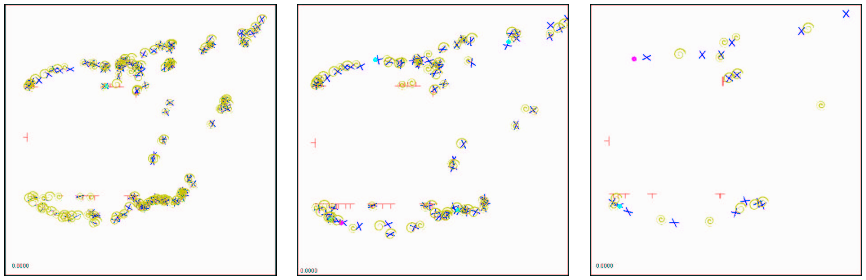

Fig. 4. Three views of the implicit method using a box filter. Left: original data set (322 critical points). Middle: 2x2 box filter (179 critical points). Right: 8x8 box filter (40 critical points).

Figure 4 illustrates the implicit method using a box filter. The left image shows the set of all 322 critical points of the original data set. The middle image shows the set of 179 critical points of the data set after being filtered with a 2x2 box filter. The right image shows the set of 40 critical points of the data set after being filtered with a 8x8 box filter.

Although implicit methods are conceptually easy to understand and may achieve the desired effect in certain cases, a number of drawbacks can be mentioned. Due to filtering, there is no direct relation between critical points in the original and derived set. The position and type of a critical point can change after the filter is applied.

3.2 Explicit methods

Explicit methods are filters which prune the set of all critical points defining the topological structure. This filtering is achieved based on knowledge of topological structure of the flow.

The *pair distance filter* (see figure 5) is a filter of this type. The idea used in this filter is that a small disturbance of the flow will result in a pair of critical points: a saddle and a non-saddle. For example, the topological structure of a two dimensional vortex consists of a focus (repelling or attracting) combined with a saddle point (see figure 5 left). The distance between the pair of critical points is an indication of the size of the vortex. Removing the critical point pair from the topology does not alter the remaining flow topology. It can be shown that a pair of critical point consisting of a saddle and a critical point with one of the other types mentioned in section 2 can be removed from the flow.

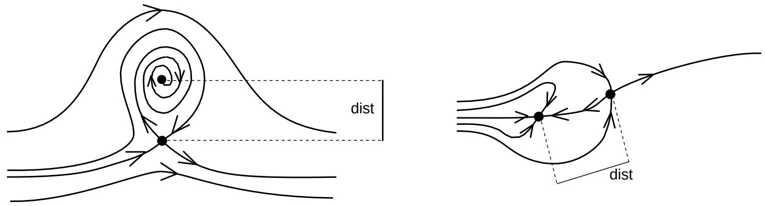

Fig. 5. Pair distance filter: Left, a focus and saddle point forming a vortex. Right, a node and saddle point. The distance is a measure for spatial scale of the disturbance of the flow.

The pair distance filter can be implemented as follows: The set of all critical points are located and classified. A distance matrix is constructed containing the distance between all possible pairs of saddles and non-saddles. The pair with the smallest distance is located and removed from the set. This is iteratively continued until all remaining pairs have a distance larger than a threshold.

Figure 6 shows the pair distance filter in practice. The left image shows the set of all 322 critical points of the original data set. The middle image shows the critical points of the data set after being filtered with a distance threshold of $0.0001 * H$, in which H is the height of the set, resulting in a set of 114 critical points. The right image shows the critical points of the data set after being filtered with a distance threshold of $0.001 * H$, resulting in a set of 34 critical points.

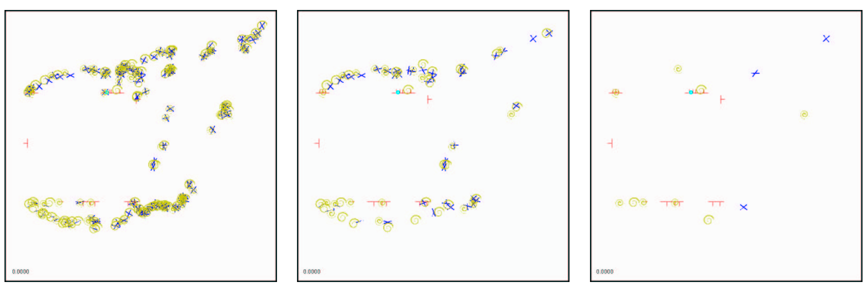

Fig. 6. Three views of the explicit method using the pair filter. Left: original data set (322 critical points). Middle: $0.0001 * H$ distance (114 critical points). Right: $0.001 * H$ filter (34 critical points).

4 Applications

4.1 Direct Numerical Simulation of Turbulent Flow

We applied our method to a turbulent flow, generated from a direct numerical simulation (DNS) by A. Veldman and R. Verstappen [6]. DNS is an accurate technique for computing turbulent flow. Flow experts use the resulting visualizations to test hypotheses about flow phenomena and – after a detailed inspection of the animation – as a means to pose new hypotheses. Of particular interest is the detailed visualization of vortex formation and the transition from laminar to turbulent flow.

In this particular problem, a DNS of a turbulent flow around a square cylinder at Re = 22,000 (at zero angle of attack) has been performed. The resolution of the rectilinear grid is 314x538x64; the grid was finest near the cylinder.

Figure 7 shows a view of the flow. This is the same 2D slice as in figure 1. The pair distance filter is used with a distance threshold of $0.001 * H$, in which H is the height of the data set. Now, streamlines can be drawn without excessive cluttering of the image while, simultaneously, maintaining the the global structure of the flow. Note, for example, the large vortex (consisting of a saddle and attracting node) behind the square cylinder.

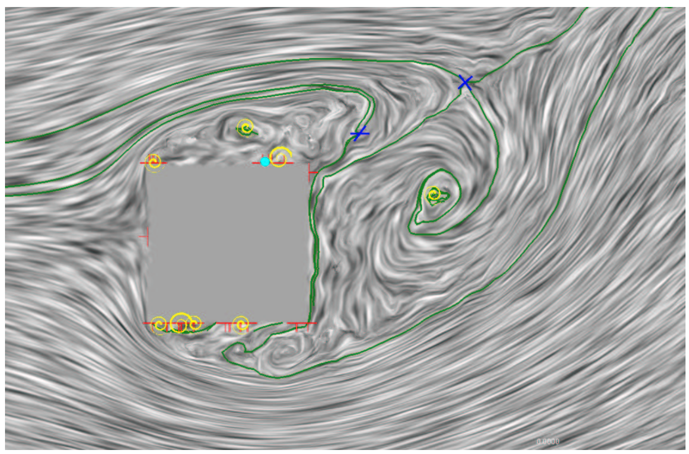

Fig. 7. A view of global flow structure around a square cylinder. The pair distance filter is used with a distance of $0.001 * H$.

Figure 8 shows a zoomed in view of the previous image. The distance threshold for the pair distance filter is adjusted to reflect structures at a smaller scale.

This application clearly benefits from the added value of the multi-level flow topology technique, [7]. The data set contains an abundance of detailed information. Using traditional topology visualization methods on these data sets, excessive cluttering can not be avoided. With the multi-level approach, simplified views of the topology can be obtained without cluttering.

The multi-level topology method has been parallelized, so that interactive rates can be obtained. The interactive multi-level topology method can be used combination with interactive spot noise [8] to realize real-time animation of the time-dependent field and interactive zooming into details one time step.

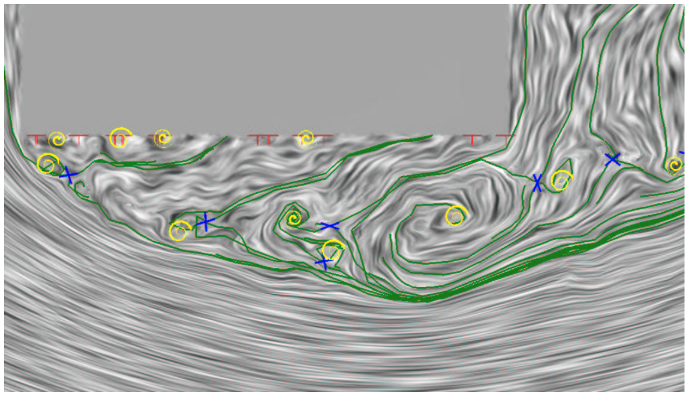

Fig. 8. A zoomed in view of flow topology around a square cylinder. The pair distance filter is used with a distance of $0.0002 * H$.

In the near future DNS can be applied to flows with a Reynolds number in the order of 10^5. The increased size and detailed information in the resulting data sets will require multi-level flow visualization techniques.

4.2 Global climate modeling.

Multi-level flow topology has also been applied to problems related to the study of atmospheric transport models. A particular topic of study is the relation between a pollutant and the wind field. For example, atmospheric pollution researchers are interested in tracking the evolution of smog and how this relates to the wind field. Non-cluttered topology depictions of wind fields are required in order to clearly display spatial scales of flow.

For this particular problem, we will only display the wind fields. The data set has been made available by the Dutch national weather institute. The data is defined on a curvilinear grid with a resolution of 144x72x16. The slice at $z = 0$ (i.e. sea level) contains 346 critical points.

Figure 9 shows two views of the wind field at $z = 0$. The pair distance filter was used to prune the critical points, resulting in a selection of 273 critical points (left) and 165 critical points (right). From the images one can clearly see various regions of flow.

Care must be taken when applying the pair distance filter to two dimensional curvilinear grids. The distance between two points is the shortest path in the surface between the two points. However, because we are interested in small distances only the 3D euclidean distance is a good estimation.

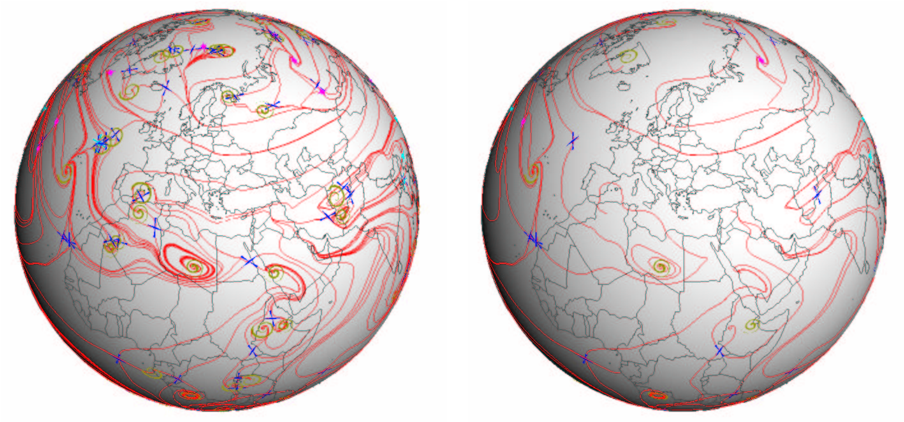

Fig. 9. Two views of a global wind field. The left image shows 273 of the original 346 critical points. Right shows 165 critical points.

5 Conclusion

In this paper, a technique for the visualization of multi-level topology in flow data sets has been presented. It provides the user with a mechanism to visualize the topology without excessive cluttering while maintaining the global structure of the flow. Two methods have been introduced. Implicit methods can be used to filter data in order to suppress flow patterns at small spatial scales. Explicit methods can be used to prune a set of critical points, making use of specific knowledge about characteristics of the flow topology. The technique has been applied to data sets resulting from a direct numerical simulation and to a global wind field.

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