Introduction to Modern Cryptography

6th lecture:

Collision-Resistant Hash Functions

quite a few of these slides are copied from or heavily inspired by the
University College London MSc InfoSec 2010 course given by Jens Groth
Thank you very much!

last time:

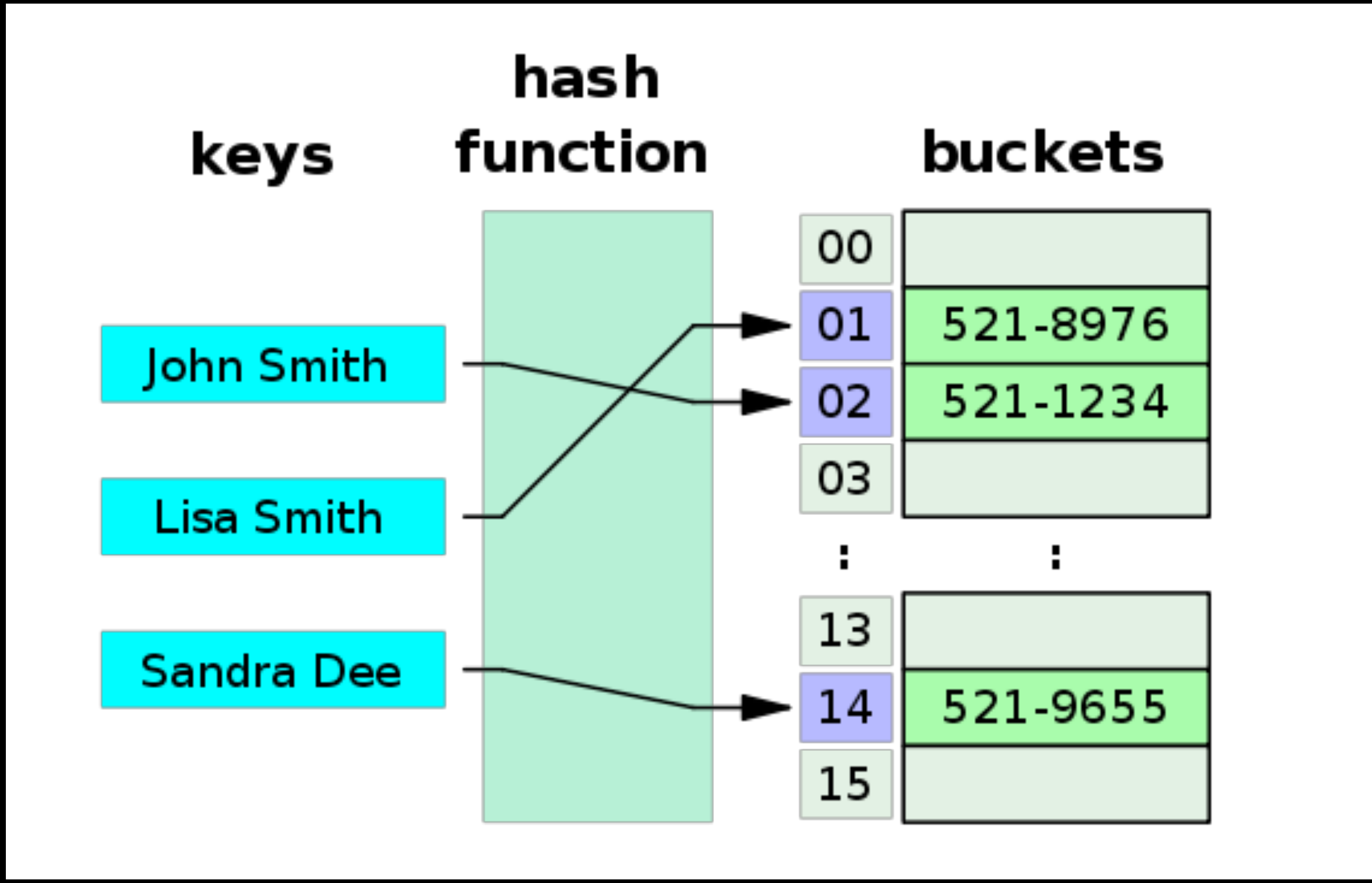
- Message Authentication Codes (MACs)
- CCA security

6th lecture (today):

- Collision-resistant hash functions

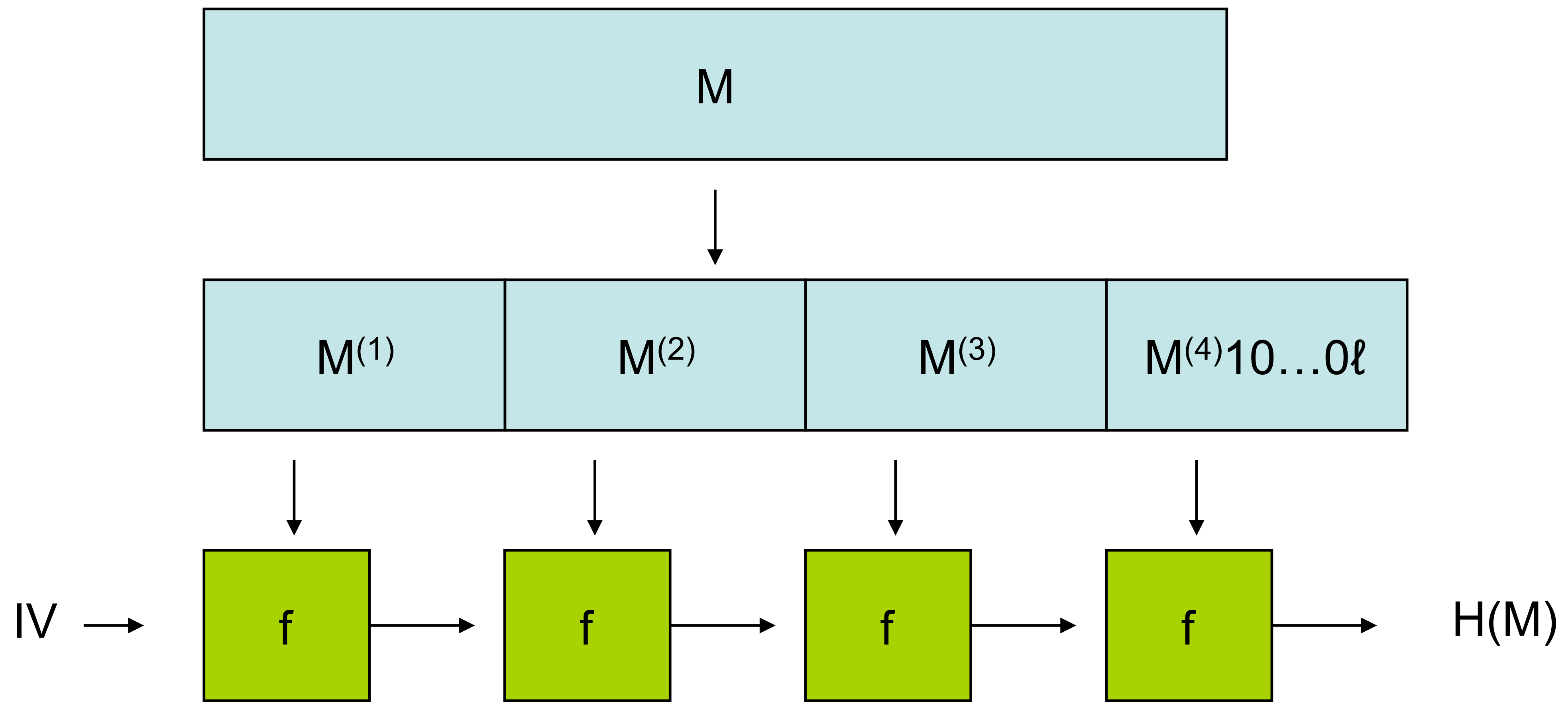
	secret key	public key
confidentiality	private-key encryption	public-key encryption
authentication	message authentication codes (MAC)	digital signatures

Hash Tables



- used in data structures to store associative arrays
- try to minimize collisions!

Merkle-Damgård Construction



Ralph C. Merkle

- co-inventor of public-key crypto
- Merkle puzzles, Merkle trees

- new ideas are hard to publish
- now interested in nanotechnology and cryonics

Ivan Bjerre Damgård

- most publishing cryptographer in the world

- my PhD advisor
- amazing person
- plays the fiddle

Birthday Attacks on Hash Functions

- In a class of N students with random birthdays $b_1, \ldots, b_N \leftarrow \{1,2,\ldots,365\}$. How large does N need to be such that
  $\Pr[\text{ exists } i \neq j : b_i = b_j \,] > 1/2$?

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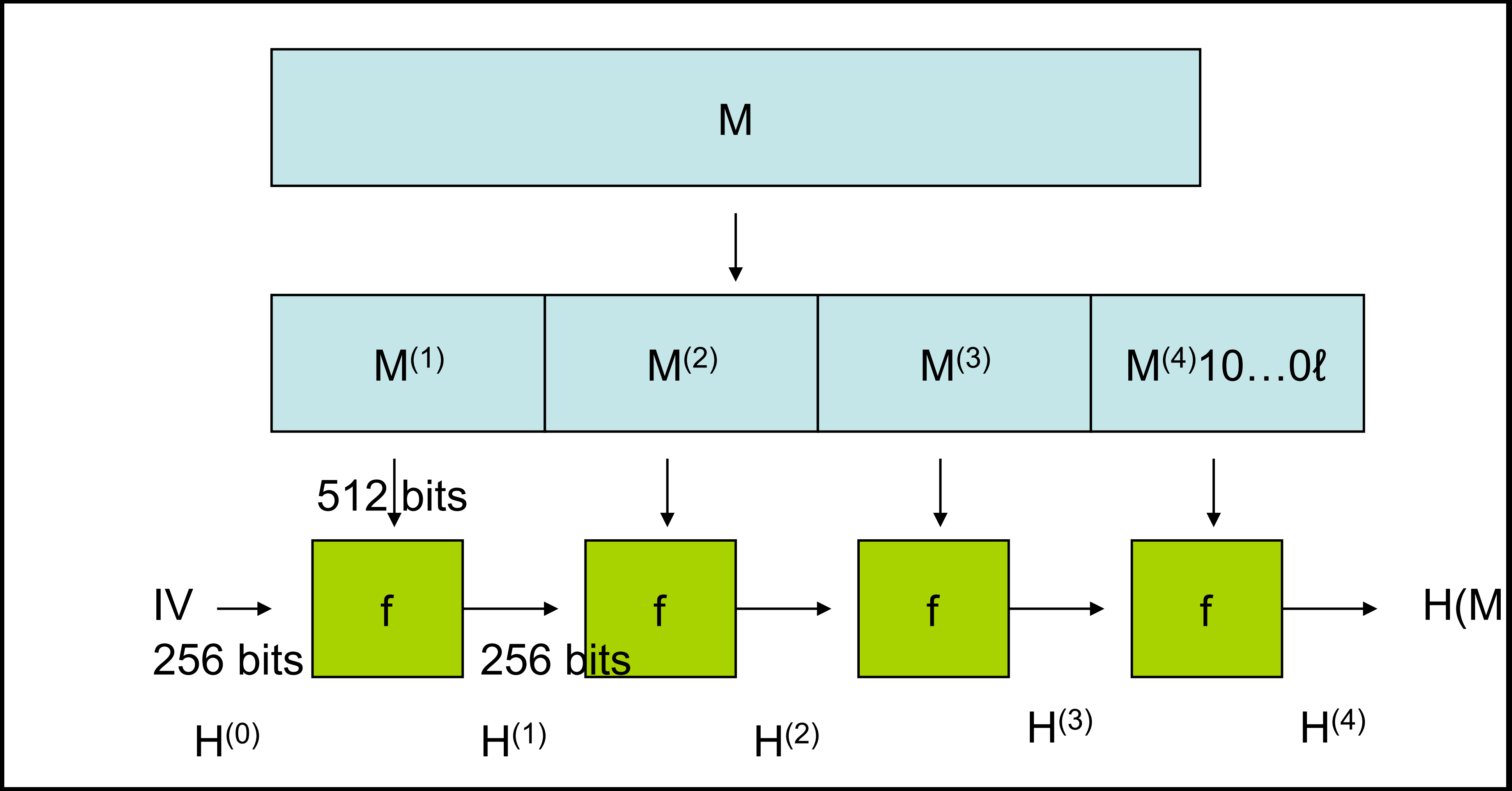
- Answer: $N \geq \sqrt{365} \approx 23$ (<u>surprisingly low</u>)
- Task: Find a generic collision-attack on the hash function $H: \{0,1\}^* \to \{0,1\}^n$

Finding Meaningful Collisions

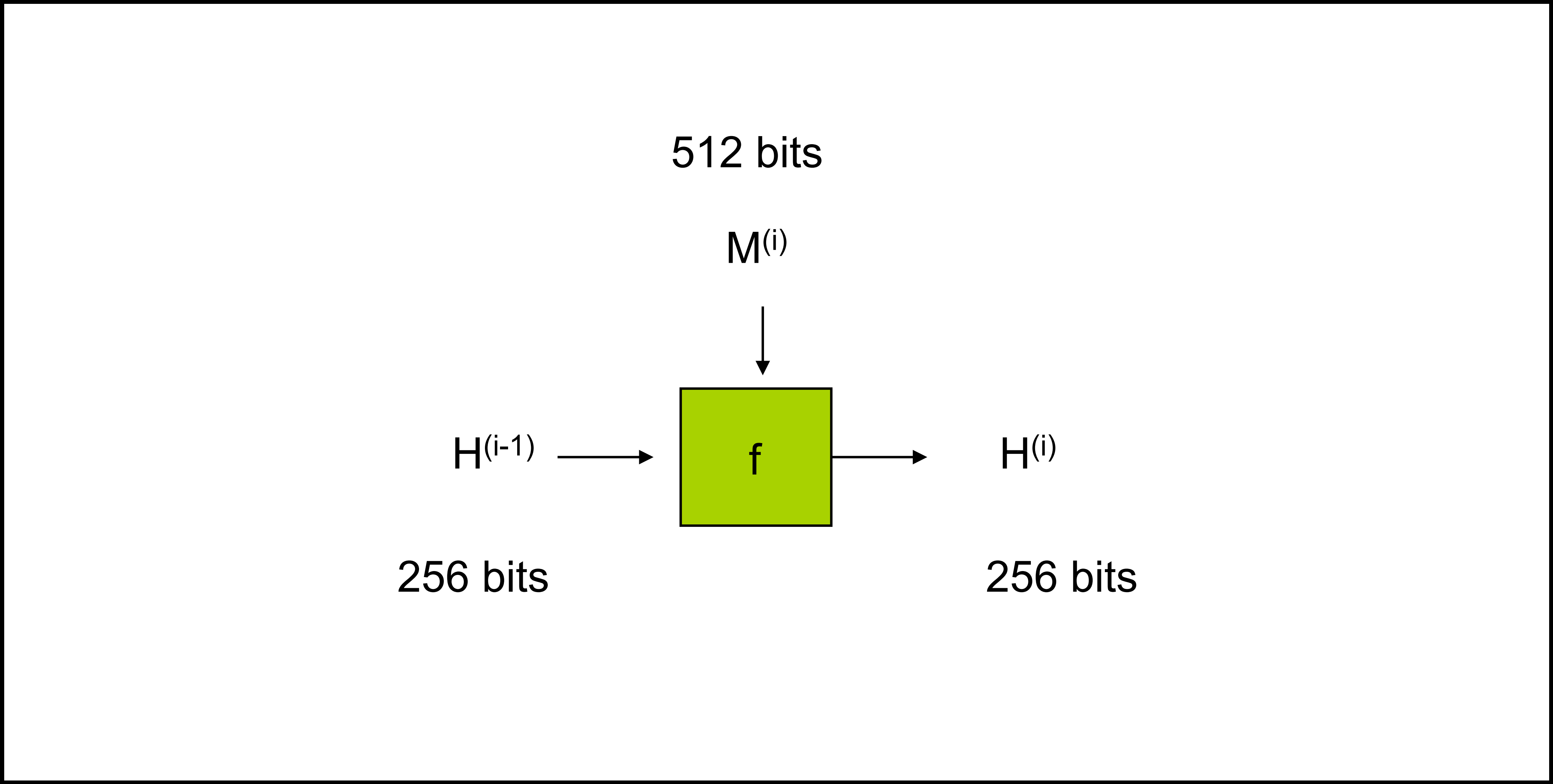
- It is {hard, difficult, challenging, impossible} to {imagine, believe} that we will {find, locate, hire} another {employee, person} having similar {abilities, skills, character} as Alice. She has done a {great, super} job
- $4 \times 2 \times 3 \times 2 \times 3 \times 2 = 288$ possibilities
- prepare $2^{n/2}$ of those and $2^{n/2}$ of bad ones

SHA-256



- detailed specification can be found here
- H(0) =`6a09e667 bb67ae85 3c6ef372 a54ff53a`
  `510e527f 9b05688c 1f83d9ab 5be0cd19`
- first 32 bits of fractional parts of square roots of 2,3,5,7,11,13,17,19

SHA-256 compression function



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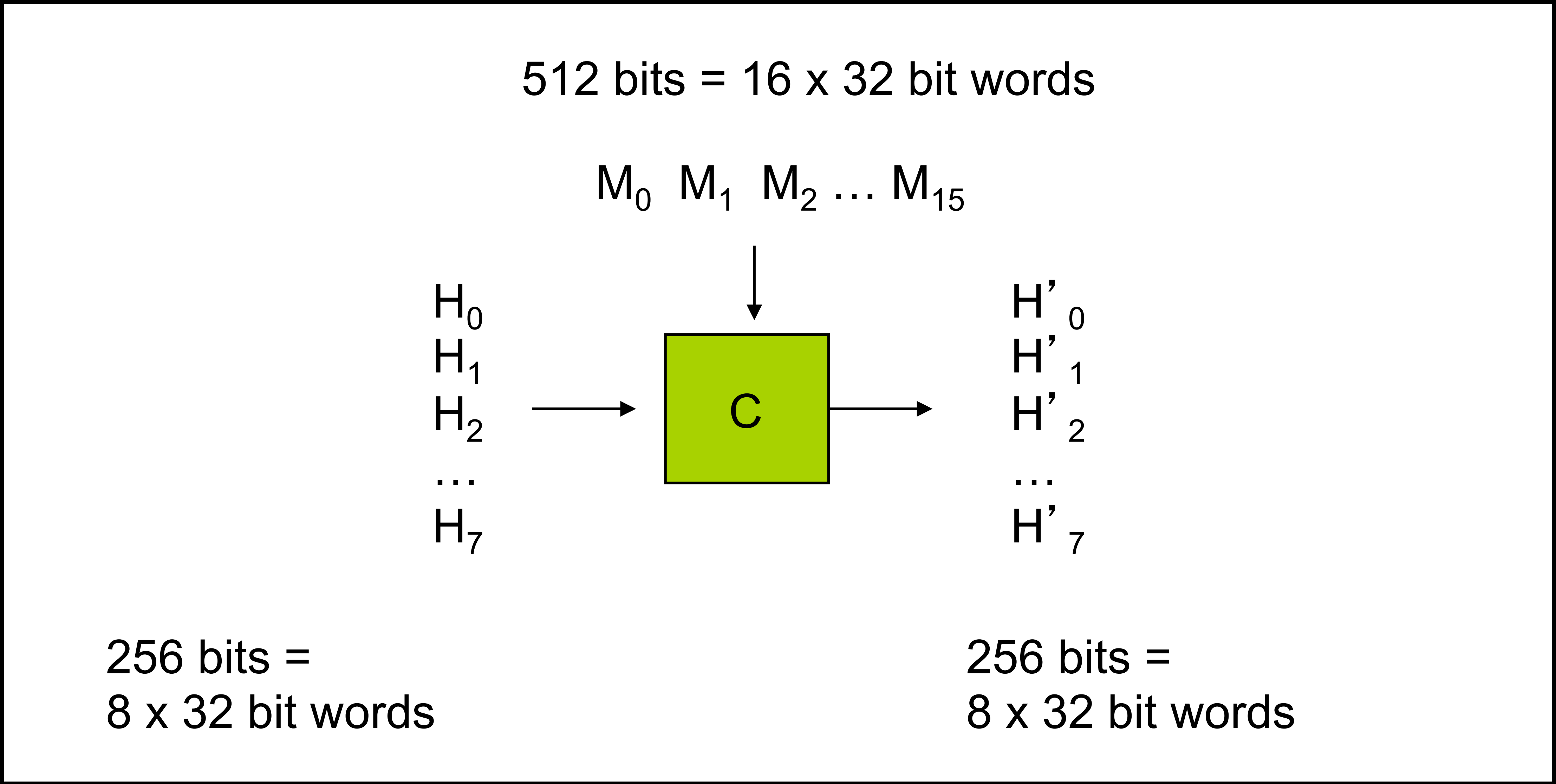

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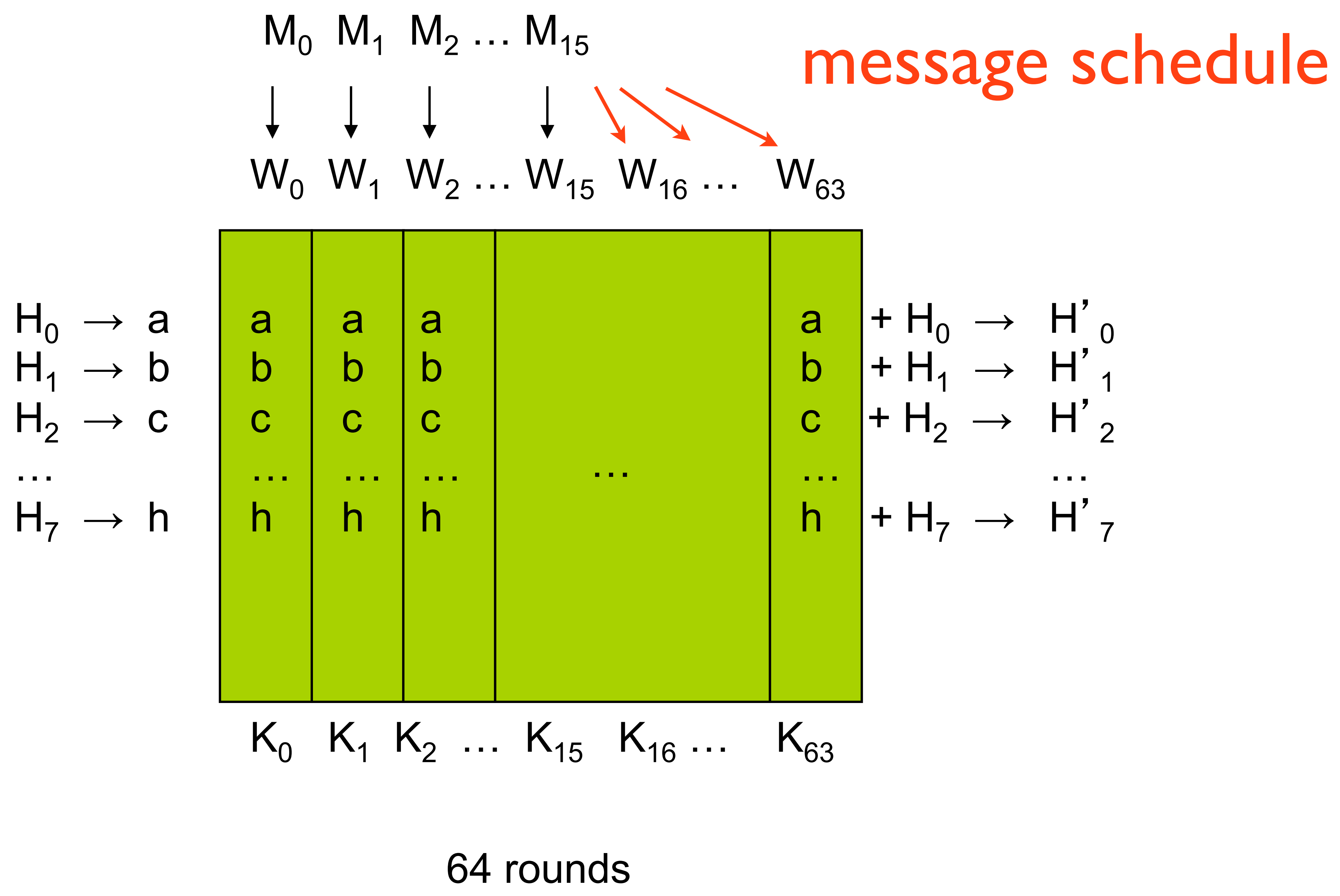
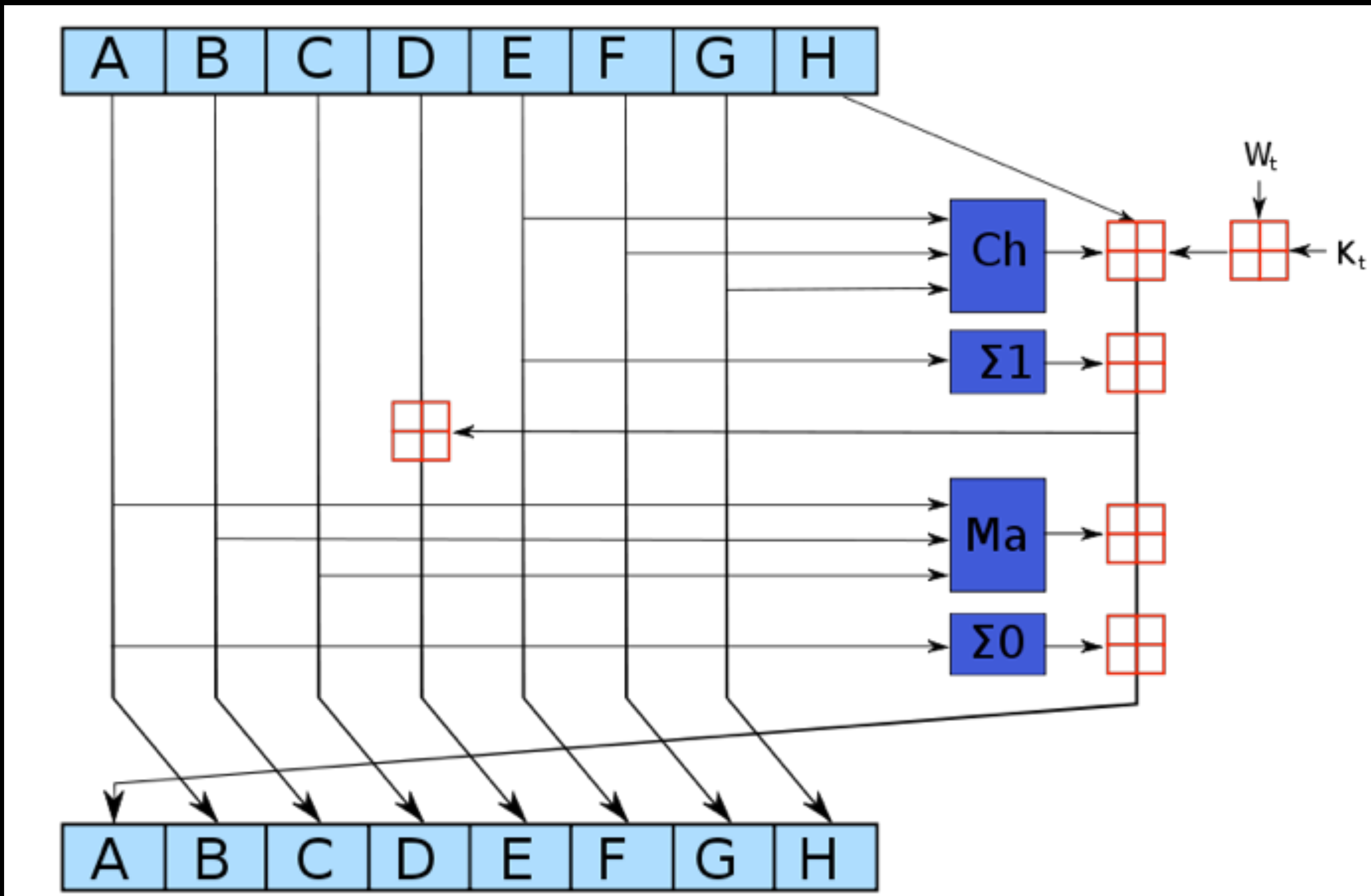


Message schedule

- $W_0 = M_0$
- …
- $W_{15} = M_{15}$
- $W_{16} = W_0 + \sigma_0(W_1) + W_9 + \sigma_1(W_{14})$
- …
- $W_{63} = W_{47} + \sigma_0(W_{48}) + W_{56} + \sigma_1(W_{61})$

- $\sigma_0(W) = ROTR^7(W) \oplus ROTR^{18}(W) \oplus SHR^3(W)$
- $\sigma_1(W) = ROTR^{17}(W) \oplus ROTR^{19}(W) \oplus SHR^{10}(W)$

Single SHA-256 round



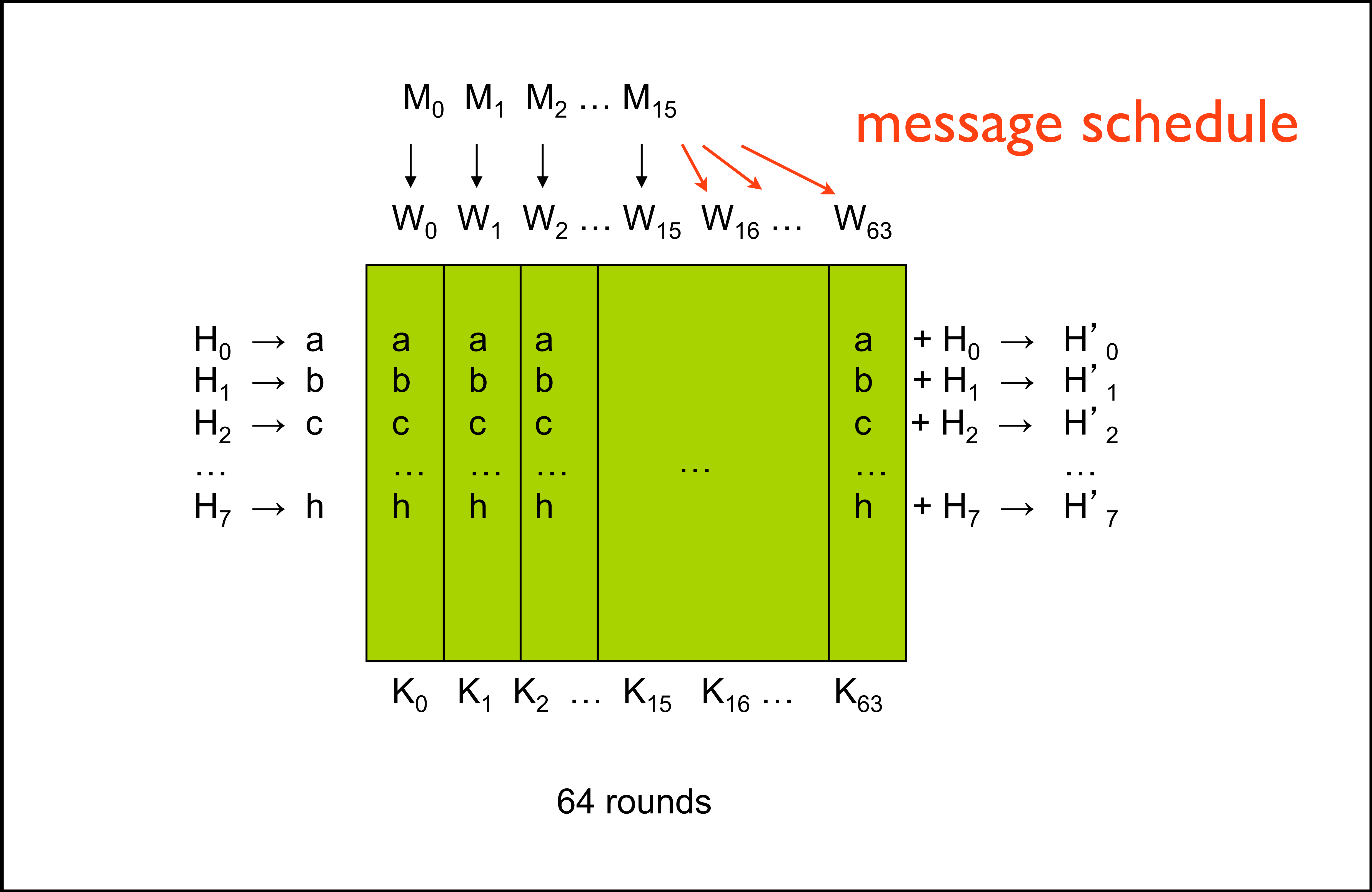
- K_0 = 428a2f98
- K_1 = 71374491
- ..
- K_{63} = c67178f2

One iteration in a SHA-2 family compression function. The blue components perform the following operations:

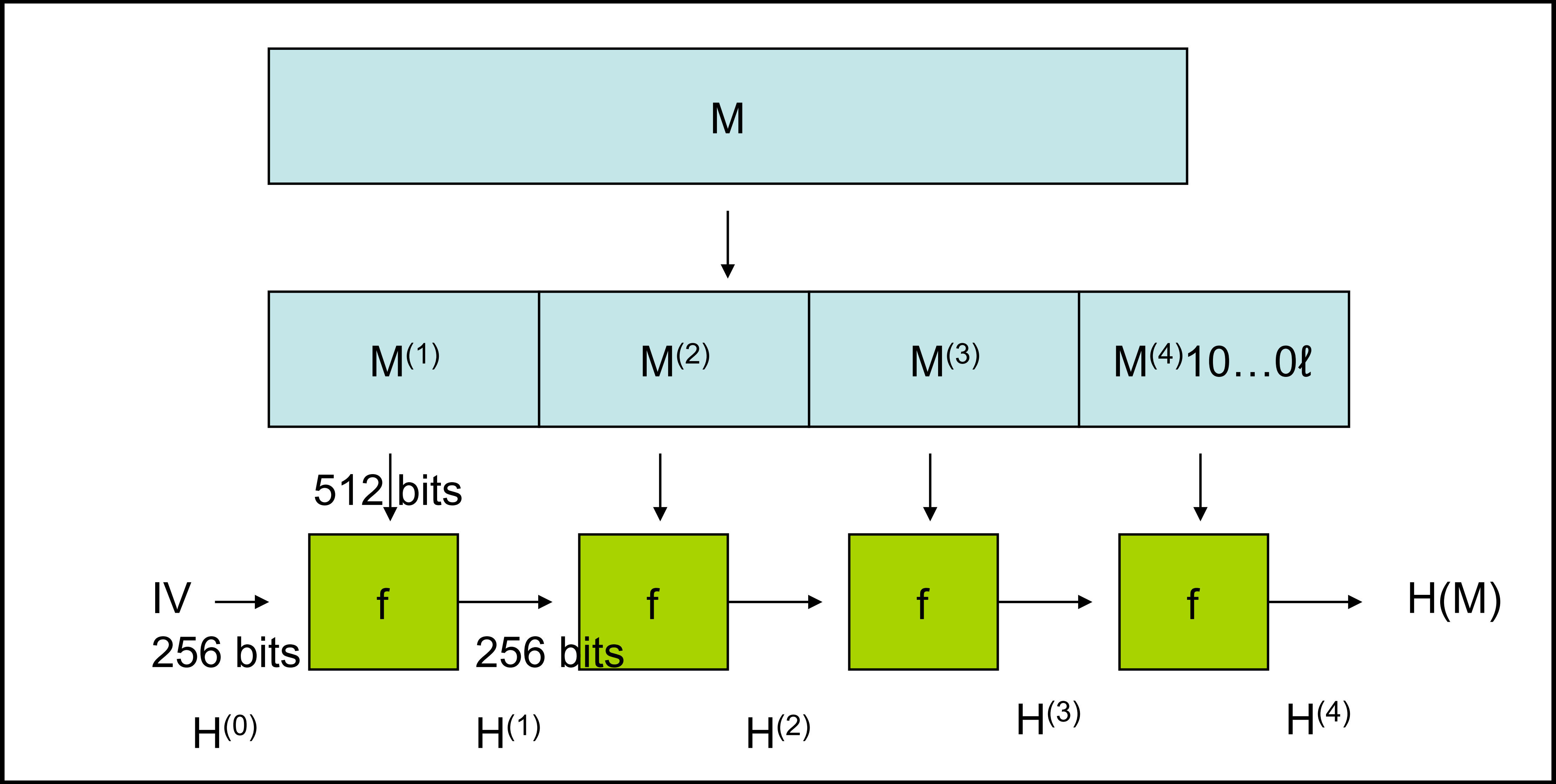
$$\mathrm{Ch}(E,F,G) = (E \land F) \oplus (\neg E \land G)$$
$$\mathrm{Ma}(A,B,C) = (A \land B) \oplus (A \land C) \oplus (B \land C)$$
$$\Sigma_0(A) = (A \ggg 2) \oplus (A \ggg 13) \oplus (A \ggg 22)$$
$$\Sigma_1(E) = (E \ggg 6) \oplus (E \ggg 11) \oplus (E \ggg 25)$$

The bitwise rotation uses different constants for SHA-512. The given numbers are for SHA-256. The red ⊞ is an addition modulo 2^{32}.

SHA-256 compression function



SHA-256



- SHA-256: $\{0,1\}^* \rightarrow \{0,1\}^{256}$ for input messages of length $|M| < 2^{64}$, i.e. M of size at most 2 billion GigaBytes
- believed to be hard to find a collision

Number Of Hashes to Find Collision

	Output bits	Birthday	Shortcut
MD4	128	2^64	2^2
RIPEMD	128	2^64	2^18
MD5	160	2^80	2^21
RIPEMD-160	160	2^80	
SHA-0	160	2^80	2^34
SHA-1	160	2^80	2^51
SHA-224	224	2^112	
SHA-256	256	2^128	

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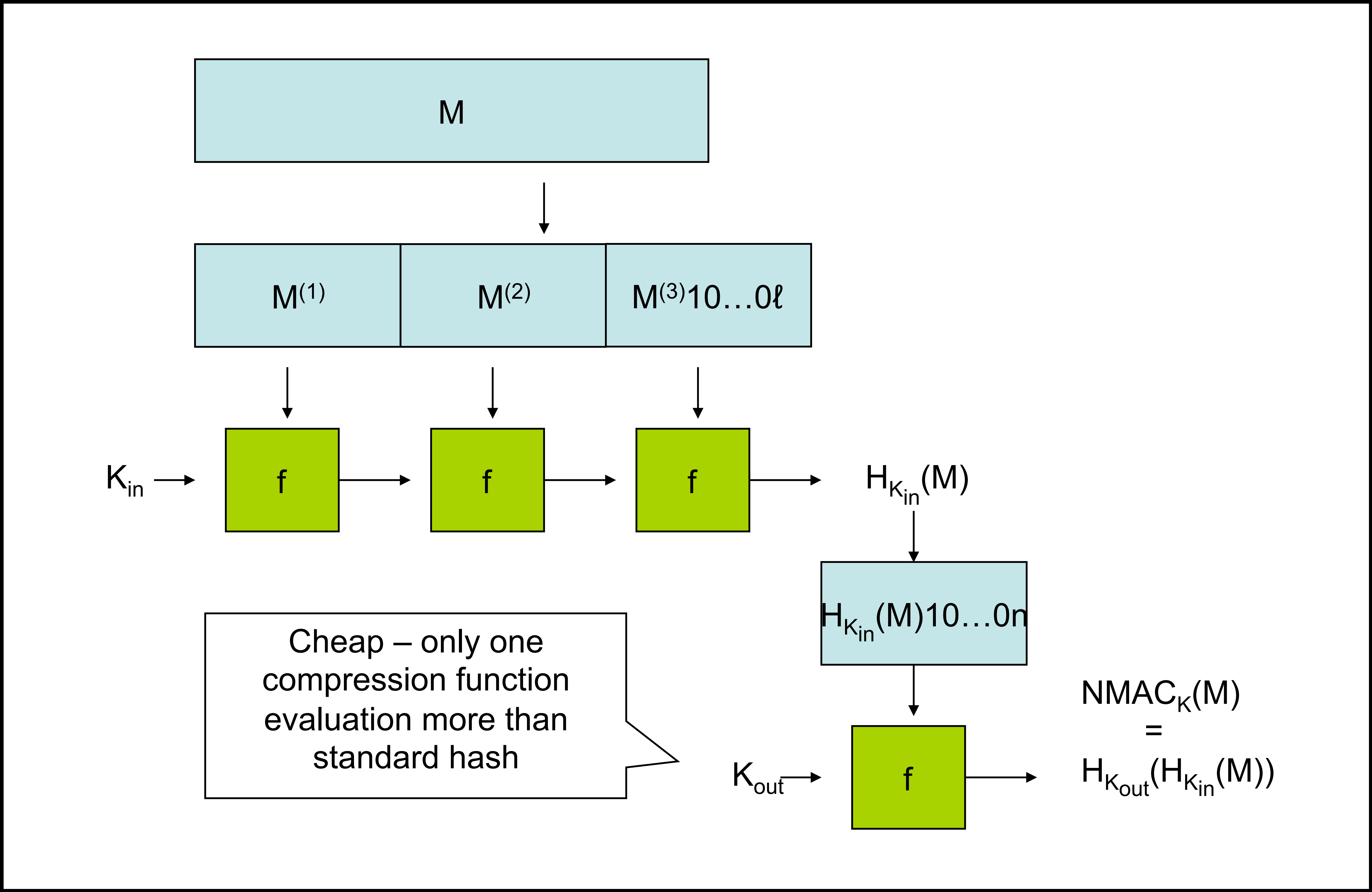
<u>creating a rogue</u>
Certification Authority
certificate

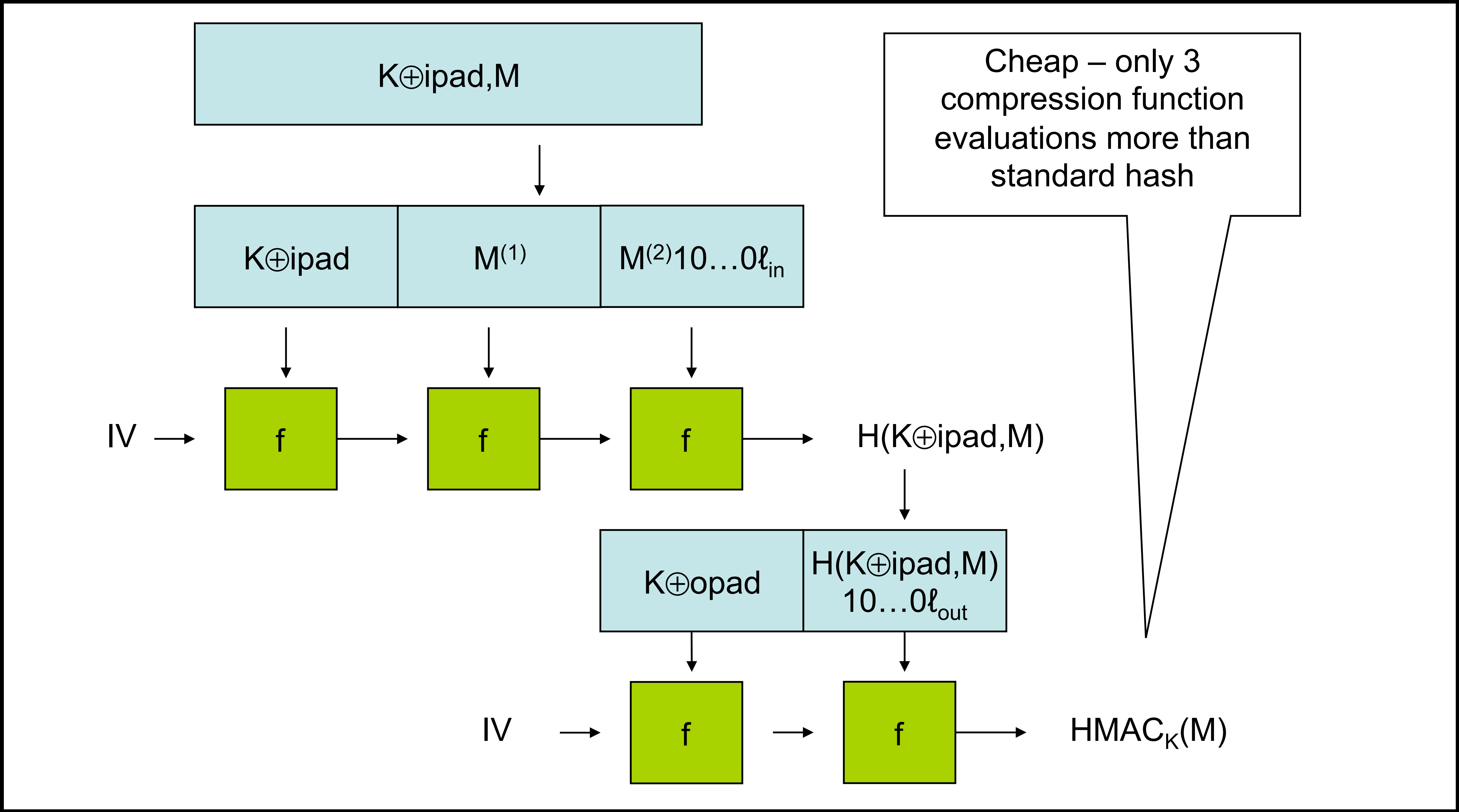
MACs from Hash Functions

NMAC-SHA-256



M
M(1)
M(2)
M(3)10…0ℓ
K_in
f
f
f
H_Kin(M)
H_Kin(M)10…0n
Cheap – only one compression function evaluation more than standard hash
K_out
f
NMAC_K(M)
=
H_Kout(H_Kin(M))

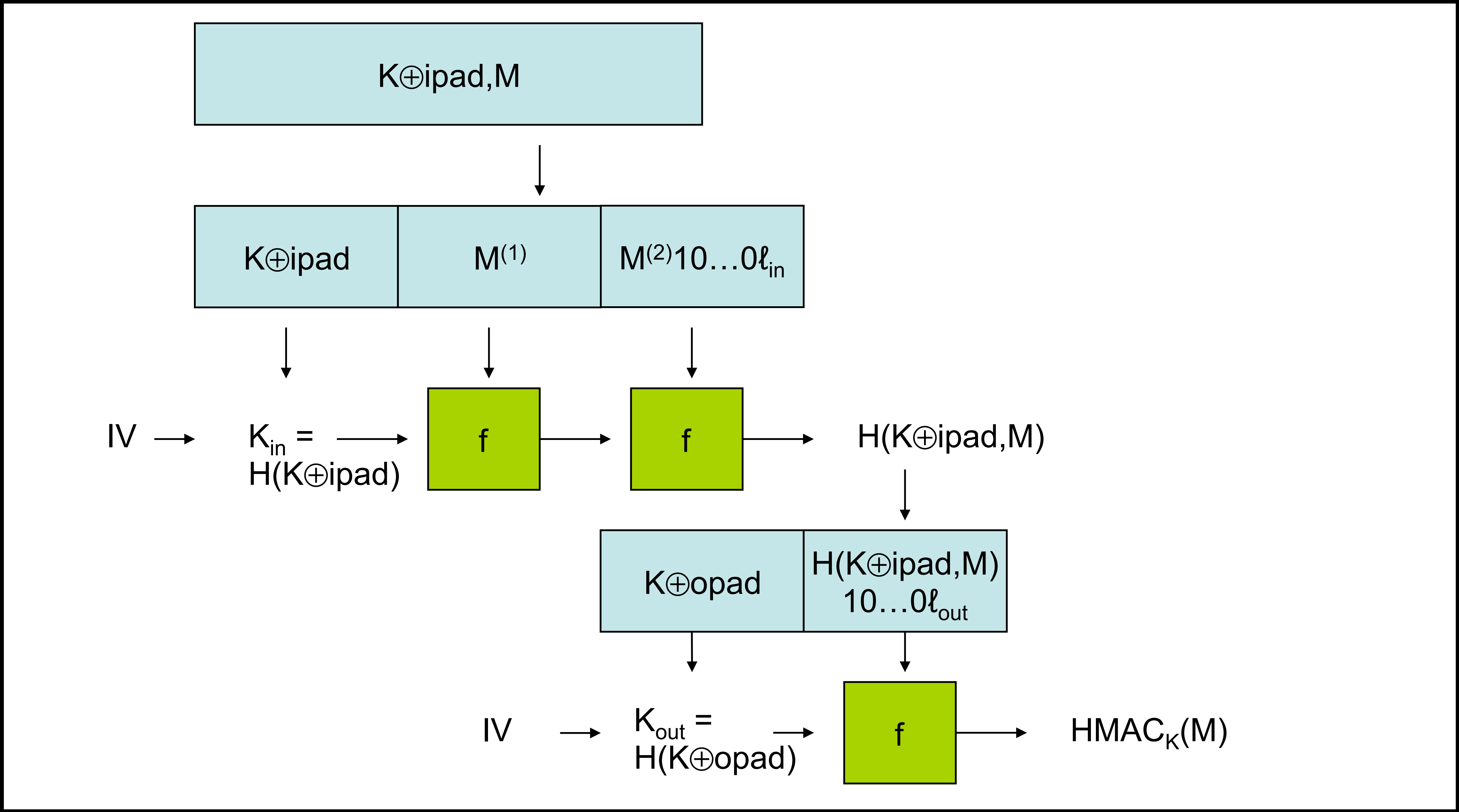
HMAC-SHA-256



- $HMAC_K(M) = H(\, K\oplus opad, H(\, K\oplus ipad\, , M\,)\,)$

  ipad = 3636...36 , opad = 5c5c...5c

HMAC vs NMAC



- HMAC can be seen as variant of NMAC
- as secure if $H(x \oplus ipad)$, $H(x \oplus opad)$ is pseudorandom