The Cramer-Shoup Cryptosystem

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October 22, 2014

The Cramer-Shoup system is an asymmetric key encryption algorithm, and was the first efficient scheme proven to be secure against adaptive chosen ciphertext attack using standard cryptographic assumptions. [2]

Outline

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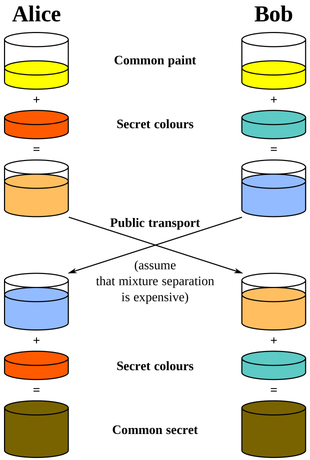
Public-key encryption



Diffie-Hellman
key exchange

`http://en.wikipedia.org/wiki/File:Diffie-Hellman_Key_Exchange.svg`

ElGamal encryption

Alice

Gen: $(q, g) \leftarrow \mathcal{G}(1^n)$

$G = \langle g \rangle$ a group, $|G| = q$

$sk = x \leftarrow \mathbb{Z}_q$

$h := g^x$

for $m \in G$: get $r \leftarrow \mathbb{Z}_q$

$\text{Enc}_{pk}(m) = (g^r, h^r m)$

$$\xrightarrow[(g^r,\, h^r m)]{pk = (g, q, h)}$$

Bob

$$\begin{aligned}\text{Dec}_{sk}(c_1, c_2) &= c_2 / c_1^x \\ &= h^r m / (g^r)^x \\ &= m\end{aligned}$$

Important results

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Decisional Diffie-Hellman Problem

$$|\Pr[\mathcal{A}(G, q, g, g^x, g^y, g^z) = 1] - \Pr[\mathcal{A}(G, q, g, g^x, g^y, g^{xy}) = 1]| \leq \mathrm{negl}(n)$$

CCA1 vs. CCA2

Malleability

An encryption algorithm is malleable if it is possible for an adversary to transform a ciphertext into another ciphertext which decrypts to a related plaintext.

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For example, in ElGamal, given (c_1, c_2) an adversary can query $(c_1, t \cdot c_2)$, which is a valid decryption for tm.

CCA1 vs. CCA2

Adaptive chosen ciphertext attacks

An interactive chosen-ciphertext attack in which the adversary sends a number of ciphertexts to be decrypted, then uses the results of these decryptions to select subsequent ciphertexts.

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$\rightarrow$ CCA2-security is equivalent to non-malleability [1]

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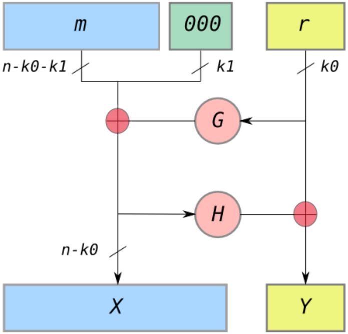
$\rightarrow$ CCA2-security is equivalent to non-malleability [1]
A CCA1-attack is also called a lunchtime attack.

Recall: OAEP for RSA



Optimal asymmetric encryption padding

http://en.wikipedia.org/wiki/File:Oaep-diagram-20080305.png

Outline

1. Motivation
   - What we've seen so far
   - Stronger notions of security
2. The Encryption Scheme
   - Cramer-Shoup
   - Proof of Security
   - Features
3. History & Implementation
   - People
   - Implementation
4. Conclusion

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Cramer-Shoup encryption

Alice

Gen: $(q, g_1, g_2) \leftarrow \mathcal{G}(1^n)$

$sk = (x_1, x_2, y_1, y_2, z) \leftarrow \mathbb{Z}_q$

$c := g_1^{x_1} g_2^{x_2}, d := g_1^{y_1} g_2^{y_2}$

$h := g_1^z$

for $m \in G$: get $r \leftarrow \mathbb{Z}_q$

$u_1 := g_1^r, u_2 := g_2^r, e := h^r m$

$\alpha := H(u_1, u_2, e), v := c^r d^{r\alpha}$

$\text{Enc}_{pk}(m) = (u_1, u_2, e, v)$

$pk = (g_1, g_2, c, d, h, H)$

$\xrightarrow{\hspace{2cm}}$

(u_1, u_2, e, v)

Bob

$\alpha := H(u_1, u_2, e)$

$$u_1^{x_1 + y_1\alpha} u_2^{x_2 + y_2\alpha} = \begin{cases} \text{verified}, & v \\ \text{abort}, & \text{otherwise} \end{cases}$$

$\text{Dec}_{sk}(u_1, u_2, e, v) = e / u_1^z$

Cramer-Shoup encryption

Correctness:

1. $u_1^{x_1+y_1\alpha} u_2^{x_2+y_2\alpha} = u_1^{x_1} u_2^{x_2} u_1^{y_1\alpha} u_2^{y_2\alpha} = g_1^{rx_1} g_2^{rx_2} g_1^{ry_1\alpha} g_2^{ry_2\alpha} = (g_1^{x_1} g_2^{x_2})^r (g_1^{y_1} g_2^{y_2})^{r\alpha} = c^r d^{r\alpha} = v$

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2. Since $u_1^z = h^r$, $\mathrm{Dec}_{sk}(u_1, u_2, e, v) = e/u_1^z = e/h^r = m$

Theorem

Cramer-Shoup is CCA2-secure

The Cramer-Shoup cryptosystem is CCA2-secure assuming that
(1) we have a universal one-way hash function H, and
(2) the Decisional Diffie-Hellman Problem is hard in the group G.

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Cramer-Shoup is CCA2-secure

The Cramer-Shoup cryptosystem is CCA2-secure assuming that
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Proof by reduction: Assuming that there is an adversary that can break the cryptosystem, and that the hash family is universal one-way, we can use this adversary to solve the Decisional Diffie-Hellman Problem.

Proof of Security

$\mathcal{S}$

$(g_1, g_2, u_1, u_2) \leftarrow \mathfrak{D}$ or $\mathfrak{R}$

$(x_1, x_2, y_1, y_2, z_1, z_2) \leftarrow \mathbb{Z}_q$

$m_0, m_1 \in G$

$c := g_1^{x_1} g_2^{x_2}, d := g_1^{y_1} g_2^{y_2}, h := g_1^{z_1} g_2^{z_2}$

$b \leftarrow \{0, 1\}$

$e := u_1^{z_1} u_2^{z_2} m_b, \alpha := H(u_1, u_2, e)$

$v = u_1^{x_1 + y_1 \alpha} u_2^{x_2 + y_2 \alpha}$

$c* = (u_1, u_2, e, v)$

$\mathcal{A}$

$\mathrm{Dec}_{sk}(c*) = e / u_1^{z_1} u_2^{z_2} = m_{b'}$

$$\begin{cases} \text{output b'}, & \mathfrak{D} \\ \perp, & \mathfrak{R} \end{cases}$$

$$\text{output} = \begin{cases} 1, & b = b' \\ 0, & b \neq b' \end{cases}$$

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- Computationally efficient, esp. when using hybrid encryption
- Intractability assumptions are minimal (only DDH & hash)

Computation

The ciphertext is about four times plaintext (not a big deal in most applications) and takes about twice as much computation as ElGamal.

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$\text{Dec}_{sk}(u_1, u_2, e, v) = e/u_1^z$

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Ronald Cramer

1968*, Dutch
Professor at the Centrum Wiskunde & Informatica (CWI) in Amsterdam and the University of Leiden
ETH Zurich, Institute for Theoretical Computer Science

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ETH Zurich, Institute for Theoretical Computer Science
hangs around in bars

Victor Shoup

born ?, USA
Professor at the Courant Institute of Mathematical Sciences (NYU)
IBM Zurich Research Laboratory

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Professor at the Courant Institute of Mathematical Sciences (NYU)
IBM Zurich Research Laboratory
on RateMyProfessors, he has an average rating of 1.4/5

Schneier on Cramer-Shoup

"If, in a few years, Cramer-Shoup still looks secure, cryptographers may look at using it instead of other defenses they are already using. But since IBM is going to patent Cramer-Shoup, probably not." [3]

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- The Cramer-Shoup system is an asymmetric key encryption algorithm based on the ElGamal scheme
- First efficient scheme proven to be secure against adaptive chosen ciphertext attacks

thank you!

References

Mihir Bellare and Amit Sahai.
Non-malleable encryption: Equivalence between two notions, and an indistinguishability-based characterization.
In *Advances in cryptology—CRYPTO'99*, pages 519–536. Springer, 1999.

Ronald Cramer and Victor Shoup.
A practical public key cryptosystem provably secure against adaptive chosen ciphertext attack.
In *Advances in Cryptology—CRYPTO'98*, pages 13–25. Springer, 1998.

Bruce Schneier.
Cramer-Shoup cryptosystem.
Crypto-Gram Newsletter, 15.09.98.