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Quantum Key Distribution

Quantum Cryptography
Motivation

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- As soon as we have quantum computers:
  - Shor's Algorithm: Integer factorisation in polynomial time!
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Can’t we obtain perfect security?
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One-Time Pad !!!
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One-Time Pad !!!
When Classical Cryptography fails...
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Quantum Cryptography

Post-quantum Cryptography
When Classical Cryptography fails...

Quantum Cryptography
  • Perfect Security
  • Relies only on the laws of nature

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Post-quantum Cryptography
- Quantum Computational Security
- Relies on primitives that are equally hard for classical and quantum computers to solve
  - Lattice-Based Cryptography
QKD 2PC and more

- Quantum Key Distribution (QKD): Two parties (Alice and Bob) communicate with perfect secrecy in the presence of an eavesdropper (Eve) [1,3,4]

- Two-Party Cooperation (2PC): Two parties that don’t trust each other cooperate in a secure way

- How to encrypt or authenticate a quantum state

- Implementations
Quantum Channel
Quantum Channel

- Eve has complete control over the channel
Quantum Channel

• Eve has complete control over the channel
• She can intercept or measure the sent qubits
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• No cloning theorem: Forbidden to clone of an unknown quantum state (Wooters and Zurek and Dieks 1982)
Quantum Channel

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- She can intercept or measure the sent qubits
- No cloning theorem: Forbidden to clone of an unknown quantum state (Wooters and Zurek and Dieks 1982)
- Eve can block the channel by sending random qubits and prevent communication over the channel
Quantum Key Distribution

• Alice and Bob use the public quantum channel to agree on a private secure key

• Eve has no information about the key

• Having a private key they can use any other classical encryption scheme to communicate through the public classical channel

• If they use OTP they can communicate with perfect secrecy!
Quantum Bits (qubits)
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- Two-state quantum-mechanical system
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- Polarisation of photon
  - rectilinear / diagonal polarization
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Measuring Qubits

NEW FROM THE CREATORS OF "PAVLOV'S DOG" IT'S...

SCHRÖDINGER'S CAT

CAT IS BOTH ALIVE AND DEAD UNTIL YOU OPEN THE BOX!

3 PAYMENTS OF $9.99

WE ARE NOT TO BE HELD RESPONSIBLE FOR THE SAFETY AND WELL-BEING OF CAT.
CAT IS NOT LEGALLY DECLARED DEAD UNTIL BOX IS OPENED. BUYER IS RESPONSIBLE FOR CAT AFTER BOX IS OPENED. NO RETURNS.
Measuring Qubits

- Measuring a qubit:
  - opening the box
Measuring Qubits

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  • opening the box

• Collapses the wavefunction to one of the two states:
  • The cat is DEAD or ALIVE
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- Collapses the wavefunction to one of the two states:
  - The cat is DEAD or ALIVE
  - The polarization is VERTICAL or HORIZONTAL
  - The bit is 0 or 1
Measuring Qubits
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1. Measuring the qubit in the “wrong basis”
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2. No information gain! (we get 0 or 1 with P=0.5)
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3. It changes the state to one of the states corresponding to the new basis
BB84 QKD Scheme

1. Alice chooses a random bit (0,1) and basis ( + or X )
2. She sends the qubit to Bob with the appropriate polarization
3. Bob measures the qubit with a random basis
4. Alice and Bob compare the string of bases they used and only keep those bits where they used the same basis
5. Error estimation and correction
6. Privacy amplification
BB84 QKD in Action

1. Alice
   - Unpolarized photon
   - Laser

2. Bob
   - Detection filter

3. Eve
   - Detection filter

### Bit Sequence

<table>
<thead>
<tr>
<th>Alice’s bit sequence:</th>
<th>0</th>
<th>0</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alice’s filter scheme:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bob’s detection scheme:</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>x</td>
<td>+</td>
<td>+</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>Bob’s bit measurements:</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Retained bit sequence (key):</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Diagram:
- Transmission of unpolarized photons from Alice to Bob.
- Eve attempts to intercept the photons.
- Detection filters at Bob and Alice for security checks.
Alice chooses a bit, basis
And sends the polarized photon to Bob
Alice chooses a bit, basis
And sends the polarized photon to Bob

Knows some bits!!
Alice chooses a bit, basis
And sends the polarized photon to Bob

Bob measures the photon with a random basis

Alice's bit sequence: 0 0 1 0 1 0 1 1 1
Alice's filter scheme: \(+\) \(+\) \(+\) \(+\) \(\times\) \(+\) \(+\) \(\times\) \(\times\)
Bob's detection scheme: \(+\) \(+\) \(+\) \(+\) \(\times\) \(+\) \(+\) \(\times\) \(\times\)
Bob's bit measurements: 1 0 1 0 1 0 0 1 1
Retained bit sequence (key): 0 0 1 – – 1 1
Alice chooses a bit, basis
And sends the polarized photon to Bob

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- Confirm bases used
- Error estimation
- Privacy Amplification

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Alice's filter scheme: + + + + + + + + + + +
Bob's detection scheme: + + + + + + + + + + +
Bob's bit measurements: 1 0 1 0 1 0 0 1 1 1
Retained bit sequence (key): 0 0 0 1 1 1 1 1
Alice chooses a bit, basis And sends the polarized photon to Bob

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This scheme can be proven to be perfectly secure!

- Confirm bases used
- Error estimation
- Privacy Amplification
Detecting an eavesdropper
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- If too many errors are detected they know that there was an eavesdropper and abort
Applications

• Commercial QKD systems already exist
• 2007 Voting in Geneva [5]
• Approximately 4 commercial companies
• and 5 Quantum Key Distribution Networks
Too good to be true?

• Distances: ~200km using optic fiber and much less through free space (air)

• Expensive equipment

• Imperfect implementations, at least two successful attacks
Conclusions

• Quantum Cryptography only relies on laws of nature
• Post-quantum cryptography relies on primitive that are difficult for quantum and classical computers
• Quantum Key Distribution allows two parties to share a key using a public quantum channel
• QKD schemes are perfectly secure, possible and work in practice although the implementation of them so far is not perfect
References

1. Quantum Cryptography, S Fehr, Foundations of Physics 2010
2. Quantum Computing, Lecture Notes, R de Wolf (Ch. 5)
3. Quantum Cryptography: Public Key Distribution and Coin Tossing, Bennett and Brassard 1984
I SURVIVED

SUCK IT, SCHRODINGER