QUANTUM HOMOMORPHIC ENCRYPTION

Christian Schaffner

(joint work with Yfke Dulek and Florian Speelman)

http://arxiv.org/abs/1603.09717

Nederlandse Organisatie voor Wetenschappelijk Onderzoek

Institute for Logic, Language and Computation (ILLC)
University of Amsterdam

QuSoft
Research Center for Quantum Software

CWI
Centrum Wiskunde & Informatica

Trustworthy Quantum Information 2016, Shanghai, China, Wednesday 29 June 2016

EXAMPLE: IMAGE TAGGING

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EXAMPLE: IMAGE TAGGING

SKYLINE

JED

EXAMPLE: IMAGE TAGGING

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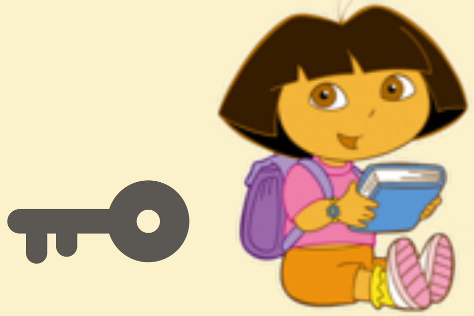
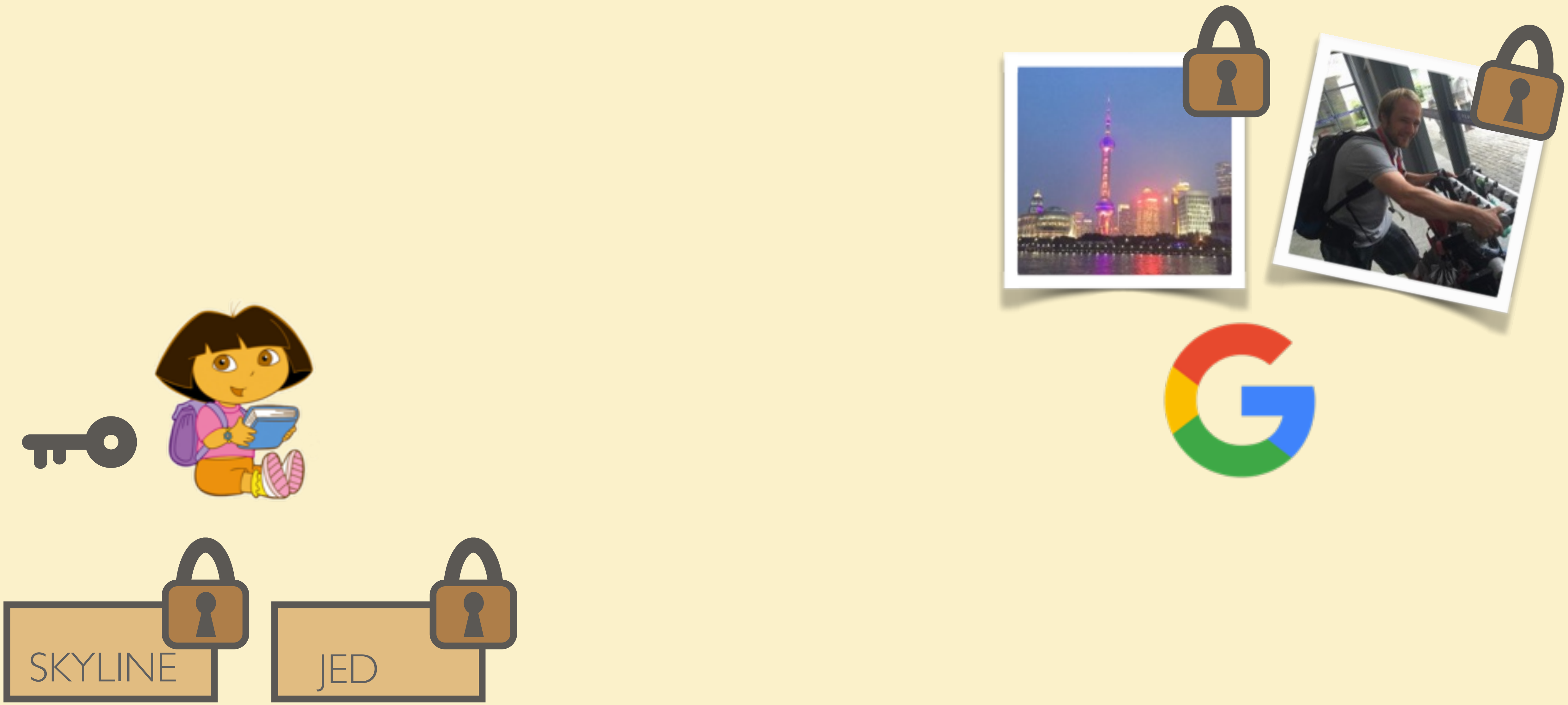


EXAMPLE: IMAGE TAGGING



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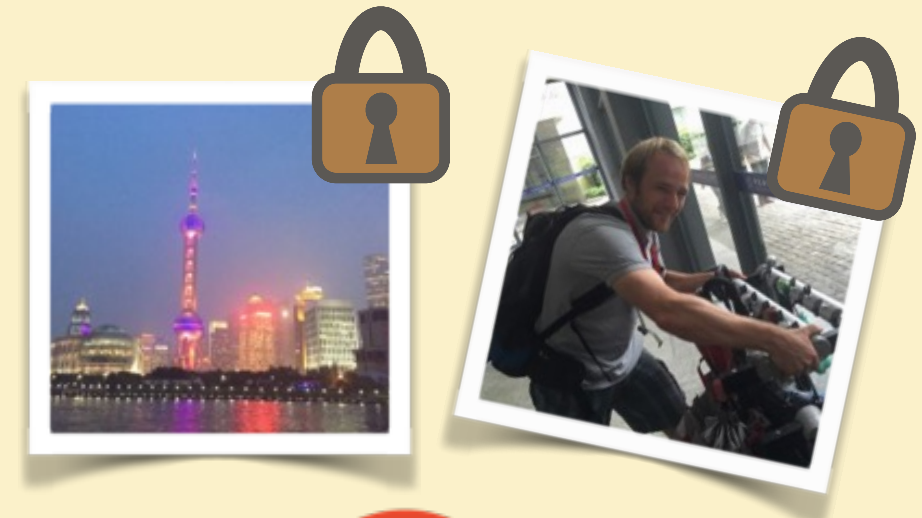
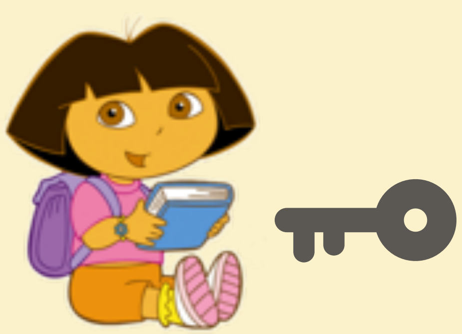




SKYLINE

JED

1. HOMOMORPHIC ENCRYPTION
2. PREVIOUS RESULTS
3. NEW RESULT

HOMOMORPHIC ENCRYPTION

HOMOMORPHIC ENCRYPTION

KEY GENERATION

HOMOMORPHIC ENCRYPTION

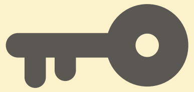
KEY GENERATION

public key

HOMOMORPHIC ENCRYPTION

KEY GENERATION

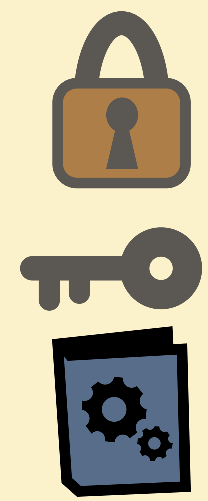
public key



secret key

HOMOMORPHIC ENCRYPTION

KEY GENERATION



public key

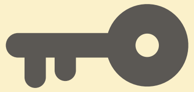
secret key

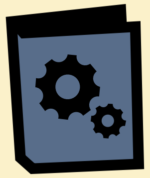
evaluation key

HOMOMORPHIC ENCRYPTION

KEY GENERATION


public key

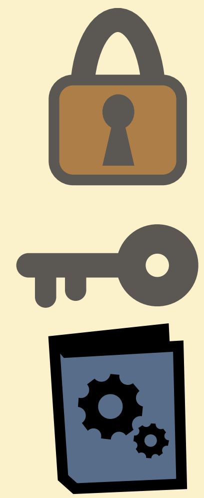

secret key


evaluation key

ENCRYPTION

HOMOMORPHIC ENCRYPTION

KEY GENERATION

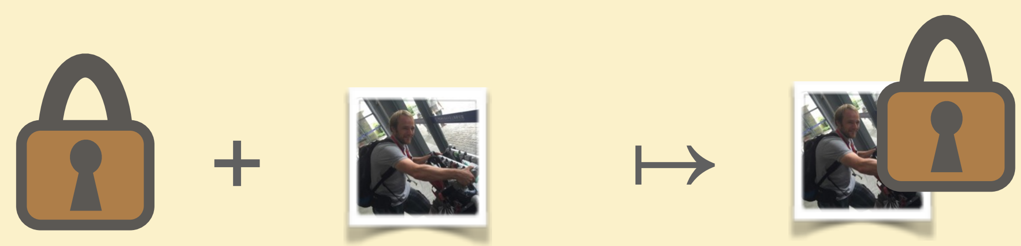


public key

secret key

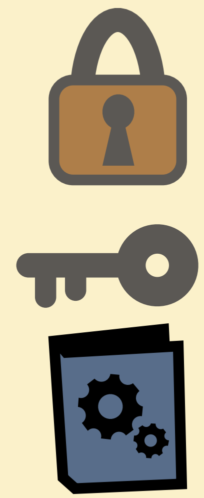
evaluation key

ENCRYPTION



HOMOMORPHIC ENCRYPTION

KEY GENERATION



public key

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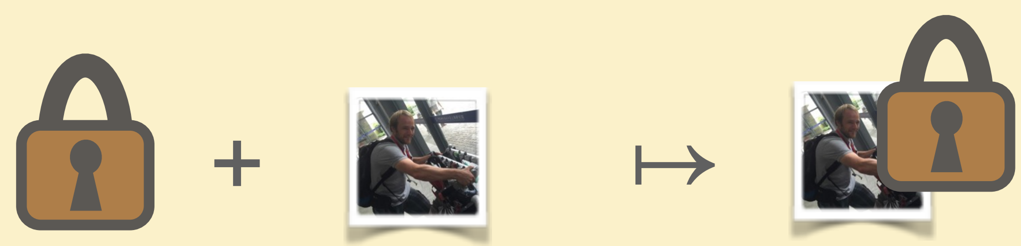
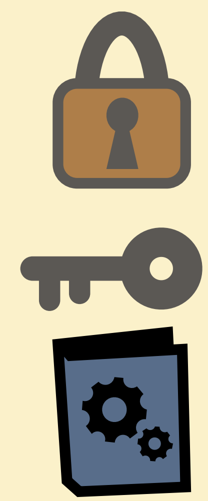
ENCRYPTION
(secure)



HOMOMORPHIC ENCRYPTION

KEY GENERATION



public key

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ENCRYPTION
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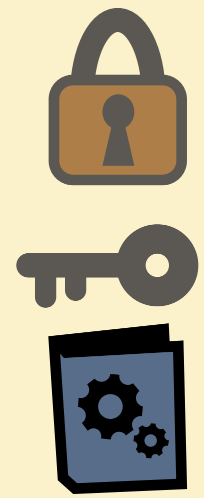
HOMOMORPHIC ENCRYPTION

KEY GENERATION



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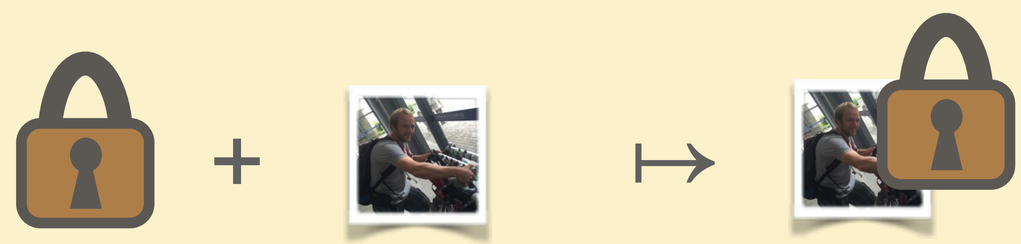
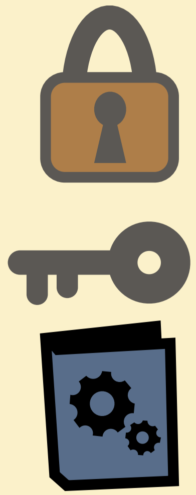


HOMOMORPHIC ENCRYPTION

KEY GENERATION

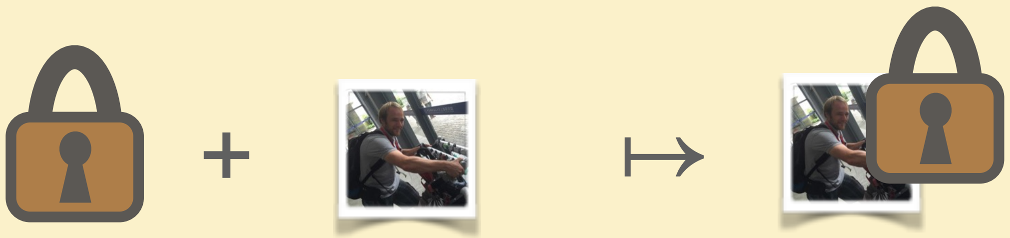


public key

secret key

evaluation key

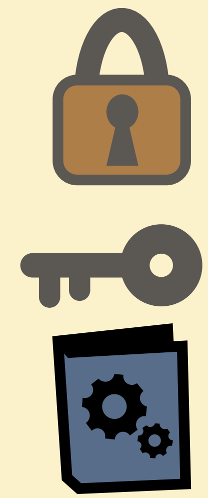
ENCRYPTION
(secure)



EVALUATION

HOMOMORPHIC ENCRYPTION

KEY GENERATION



public key

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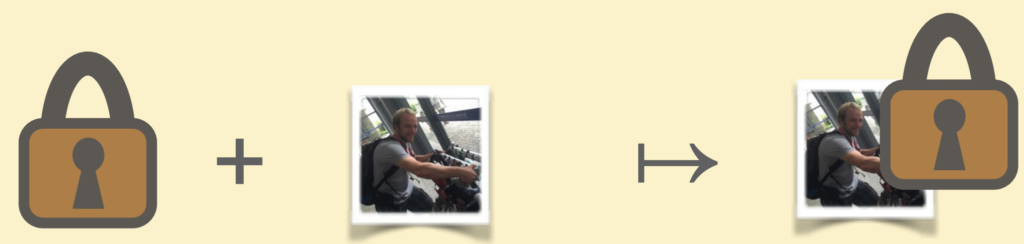
ENCRYPTION
(secure)



EVALUATION

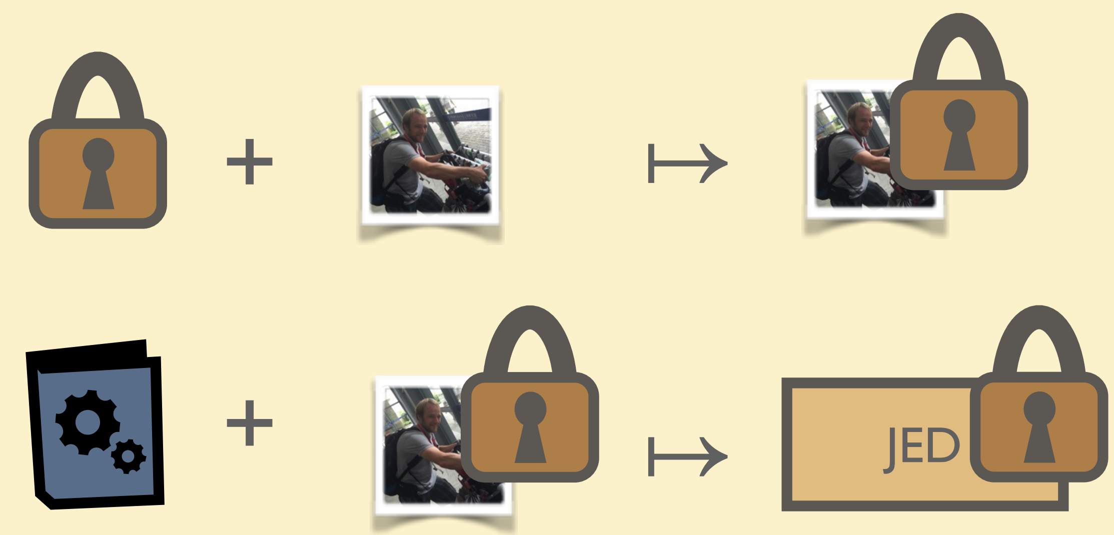
HOMOMORPHIC ENCRYPTION

KEY GENERATION

- public key
- secret key
- evaluation key

ENCRYPTION
(secure)

EVALUATION



DECRYPTION

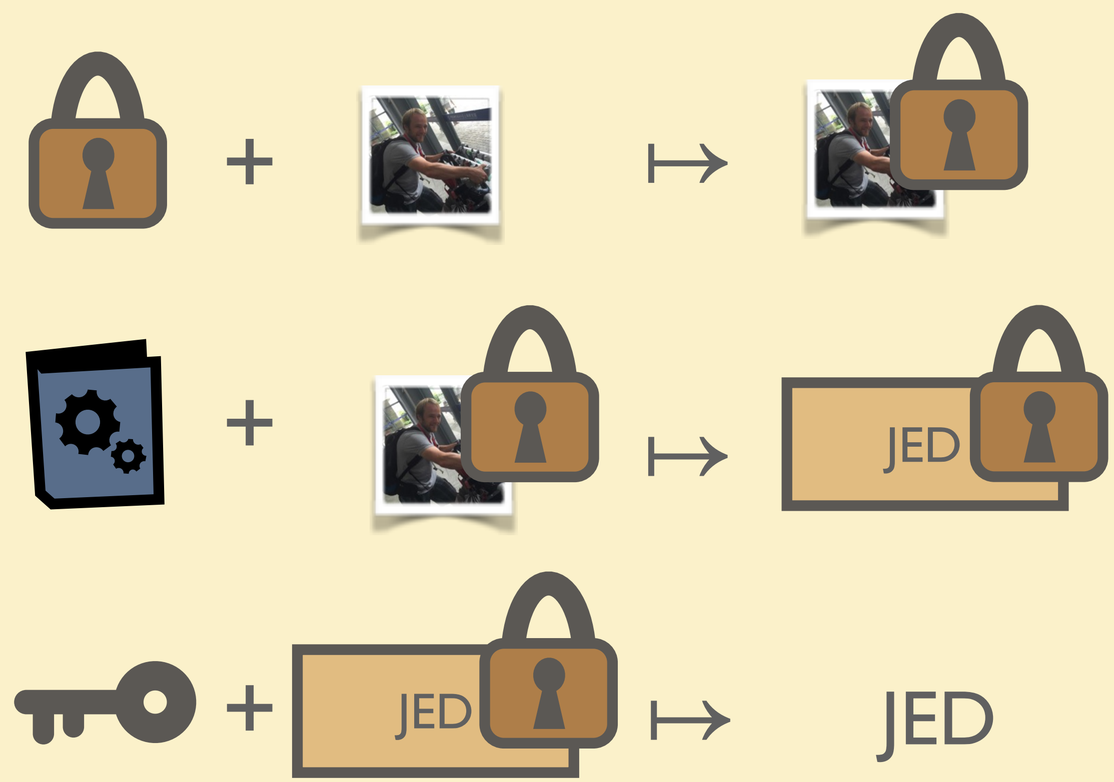
HOMOMORPHIC ENCRYPTION

KEY GENERATION
- public key
- secret key
- evaluation key

ENCRYPTION (secure)

EVALUATION

DECRYPTION



HOMOMORPHIC ENCRYPTION

KEY GENERATION		public key secret key evaluation key
ENCRYPTION (secure)		public key + x ↦ [x]
EVALUATION		evaluation key + [x] ↦ [f(x)]
DECRYPTION		secret key + [f(x)] ↦ f(x)

HOMOMORPHIC ENCRYPTION

KEY GENERATION

public key

secret key

evaluation key

ENCRYPTION (secure)

+ $|\psi\rangle \mapsto$ $|\psi\rangle$

EVALUATION

+ $|\psi\rangle \mapsto$ $U|\psi\rangle$

DECRYPTION

+ $U|\psi\rangle \mapsto U|\psi\rangle$

HOMOMORPHIC ENCRYPTION

KEY GENERATION

ENCRYPTION
(secure)

EVALUATION

DECRYPTION

✓ HOMOMORPHIC ENCRYPTION

2. PREVIOUS RESULTS

3. NEW RESULT

PREVIOUS RESULTS: OVERVIEW

C. Gentry: Fully homomorphic encryption using ideal lattices. STOC’09
A. Broadbent, S. Jeffery. Quantum Homomorphic Encryption for Circuits of Low T-gate Complexity. CRYPTO 2015
Y. Ouyang, S-H. Tan, J. Fitzsimons. Quantum homomorphic encryption from quantum codes. arxiv:1508.00938

PREVIOUS RESULTS: OVERVIEW

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  - Clifford scheme allowing evaluation of {P, H, CNOT}
  - schemes for {P, H, CNOT} + limited # of T gates

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SCHEME FOR {P, H, CNOT}

[AMTW00] A. Ambainis, M. Mosca, A. Tapp, and R. De Wolf. Private quantum channels. FOCS'00
[Gentry 09] C. Gentry: Fully homomorphic encryption using ideal lattices. STOC'09

SCHEME FOR {P, H, CNOT}

Ingredient 1: quantum encryption (one-time pad)

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SCHEME FOR {P, H, CNOT}

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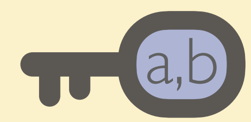
encryption: pick a,b $\in_R$ {0,1} 

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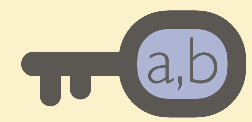
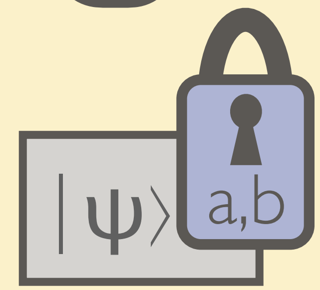


$$|\psi\rangle \mapsto X^a Z^b |\psi\rangle =$$



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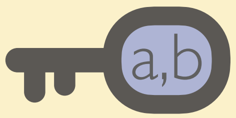

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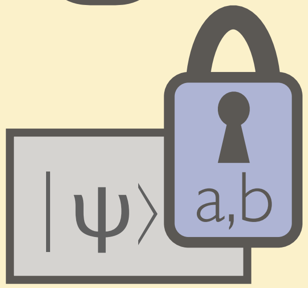

decryption:

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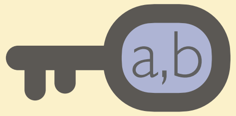

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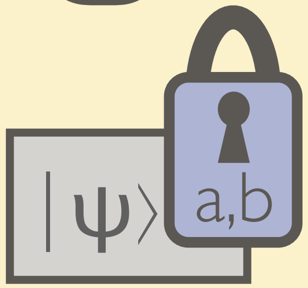

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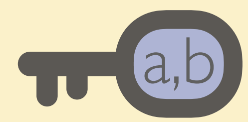

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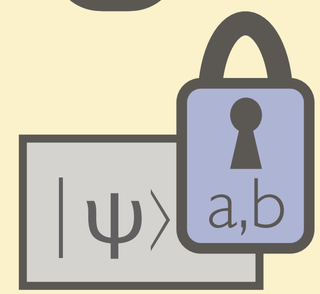

decryption: $X^a Z^b |\psi\rangle \mapsto |\psi\rangle$

Ingredient 2: classical homomorphic encryption

[AMTW00] A. Ambainis, M. Mosca, A. Tapp, and R. De Wolf. Private quantum channels. FOCS'00
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SCHEME FOR {P, H, CNOT}

Folklore, last formalized by [BJ15] A. Broadbent, S. Jeffery. Quantum Homomorphic Encryption for Circuits of Low T-gate Complexity. CRYPTO 2015

SCHEME FOR {P, H, CNOT}

$|\psi\rangle$

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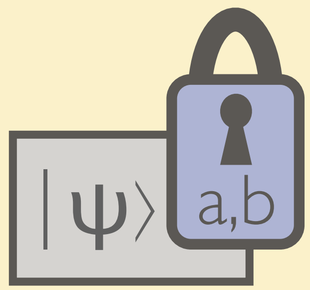
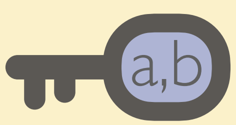
SCHEME FOR {P, H, CNOT}





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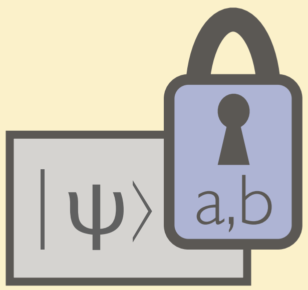

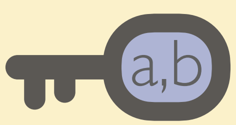

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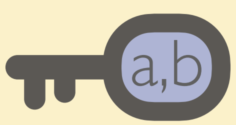

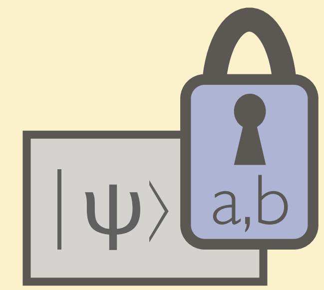

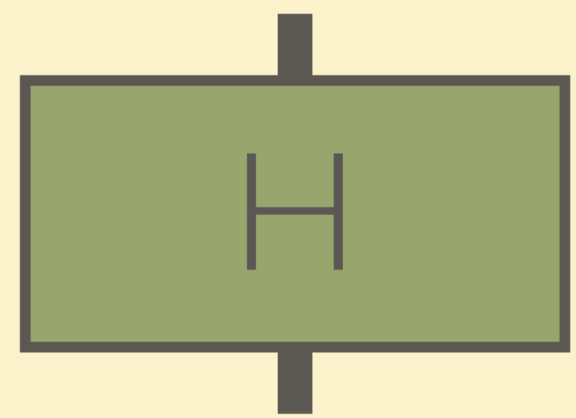

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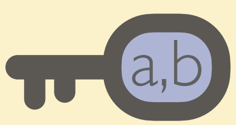

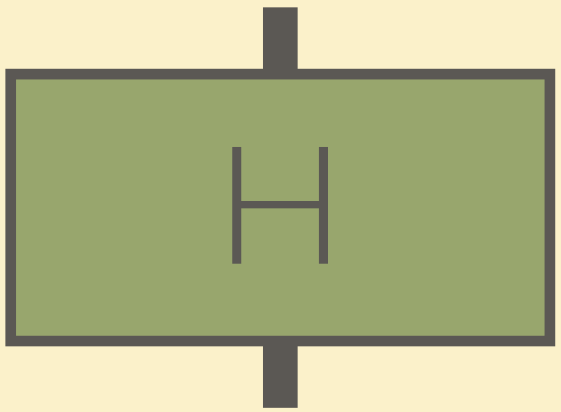

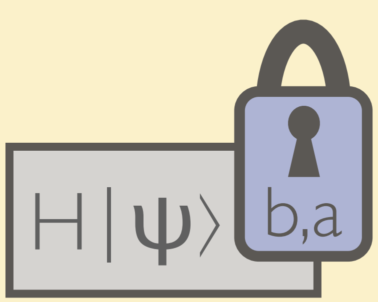

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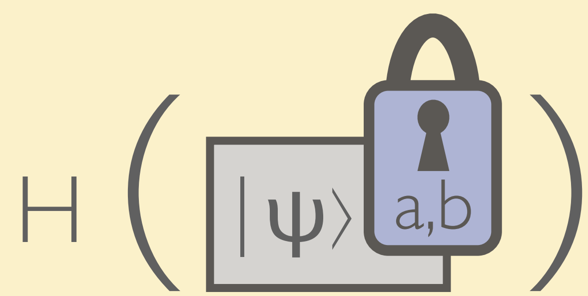
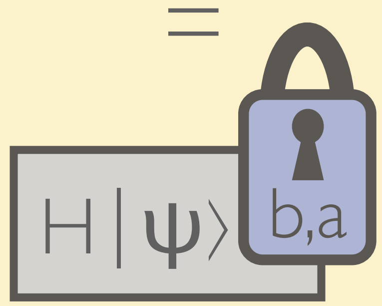
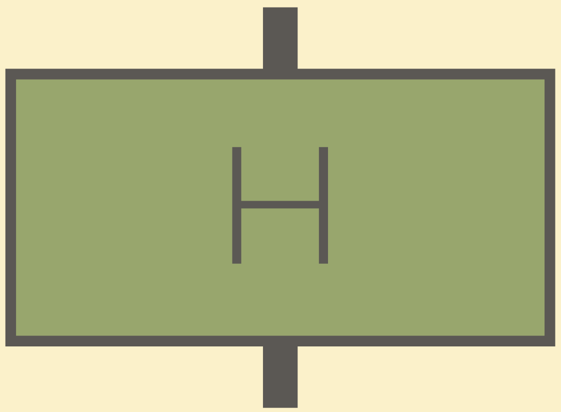
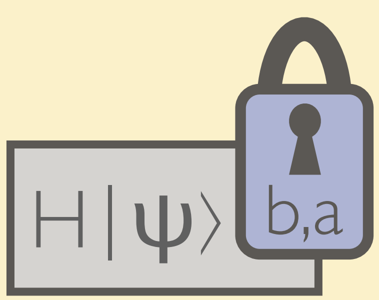


$$= HX^aZ^b|\psi\rangle = X^bZ^aH|\psi\rangle =$$







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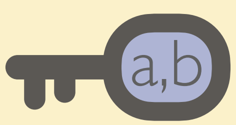

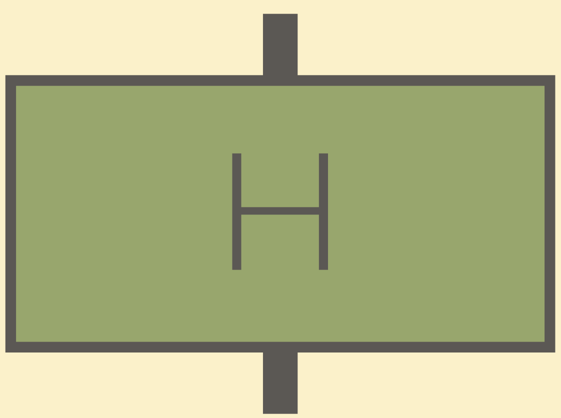

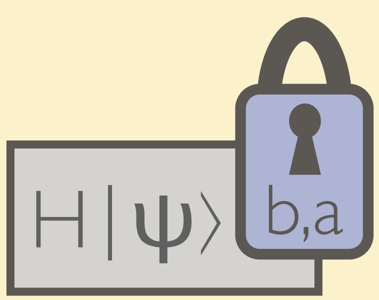

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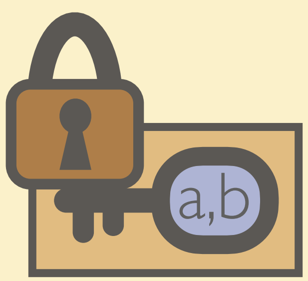
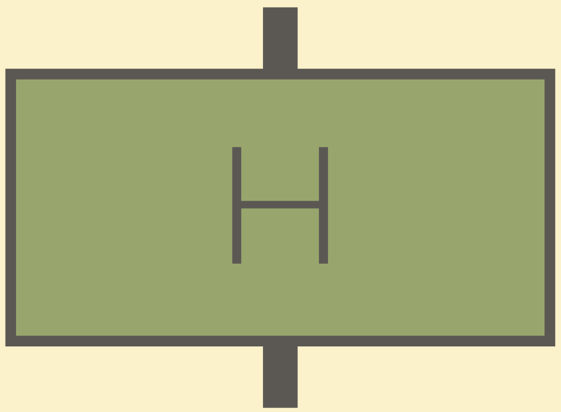
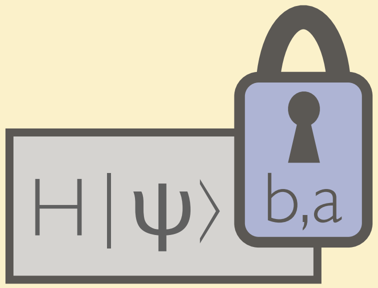
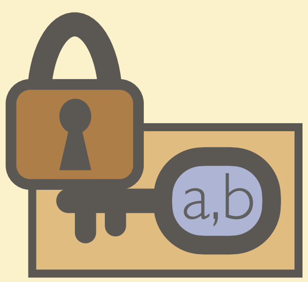


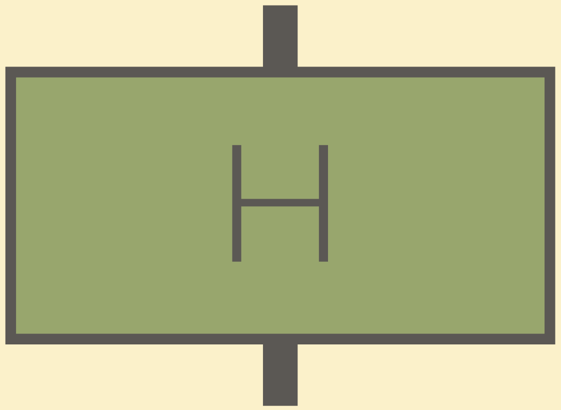


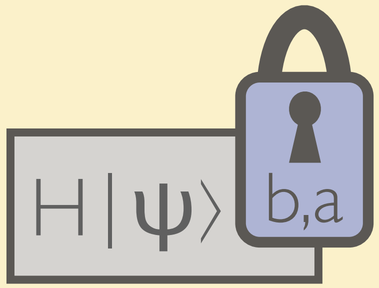


Folklore, last formalized by [BJ15] A. Broadbent, S. Jeffery. Quantum Homomorphic Encryption for Circuits of Low T-gate Complexity. CRYPTO 2015

SCHEME FOR {P, H, CNOT}







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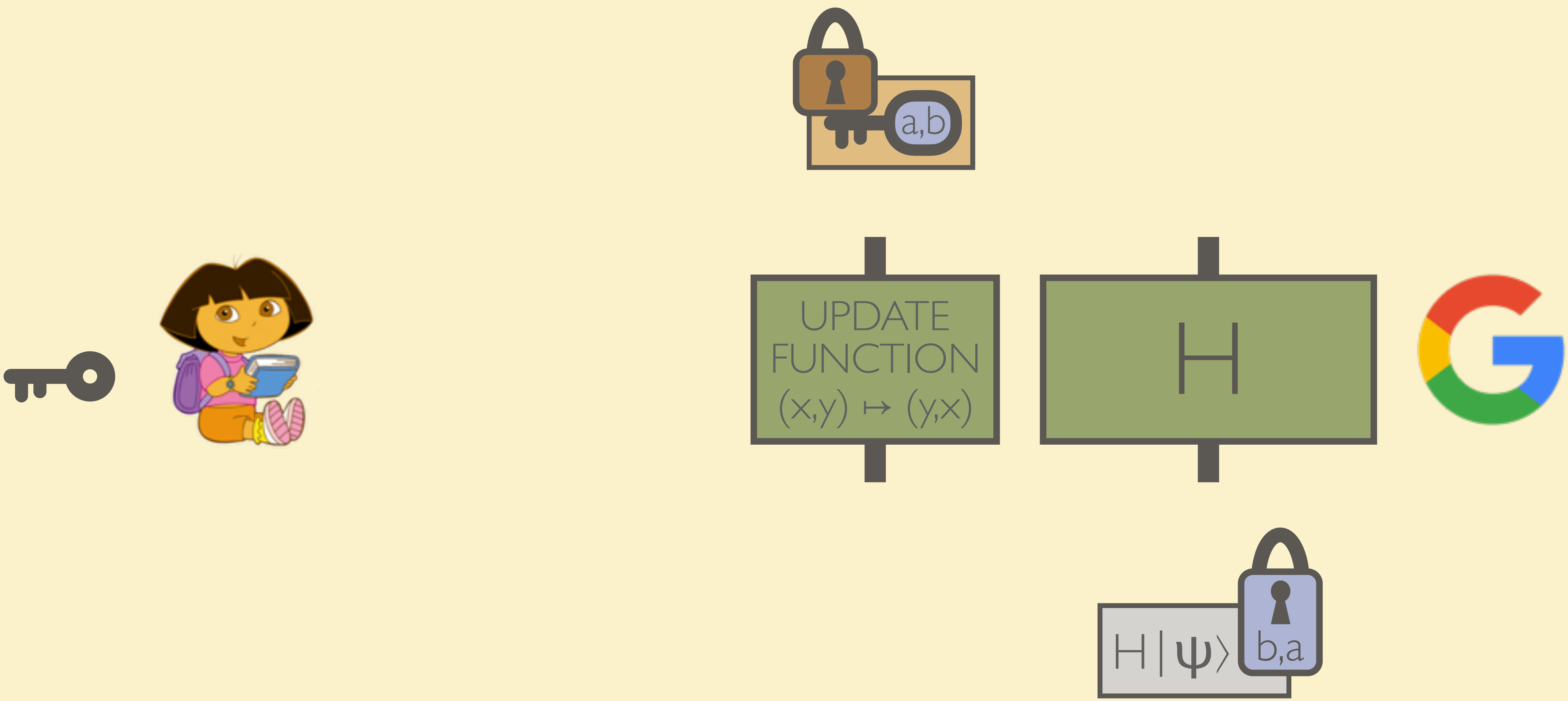

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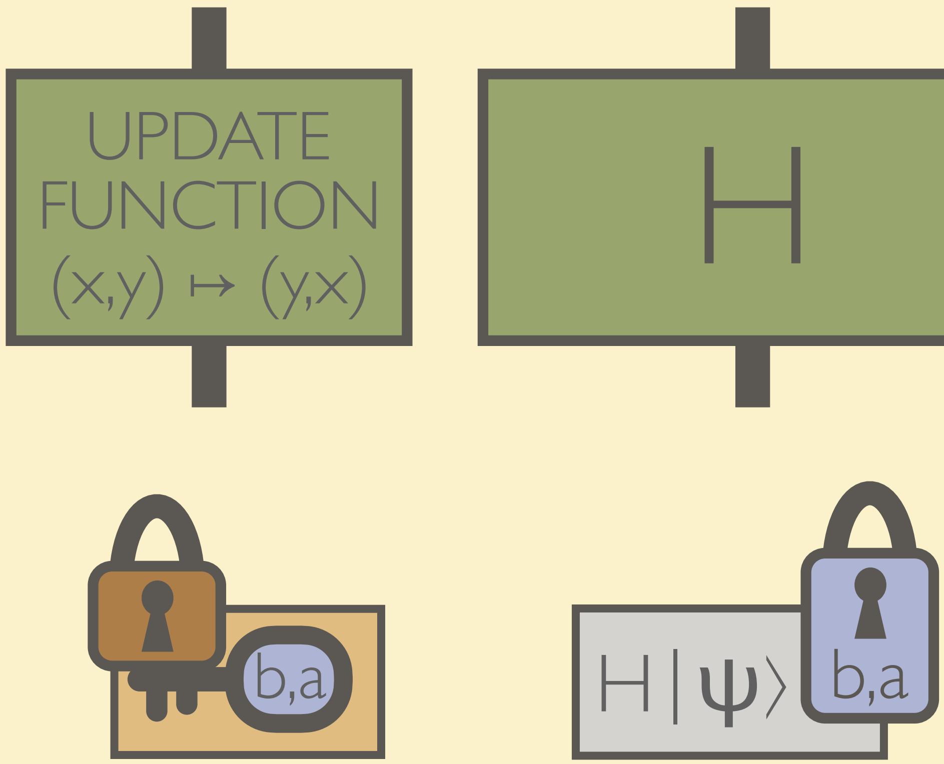

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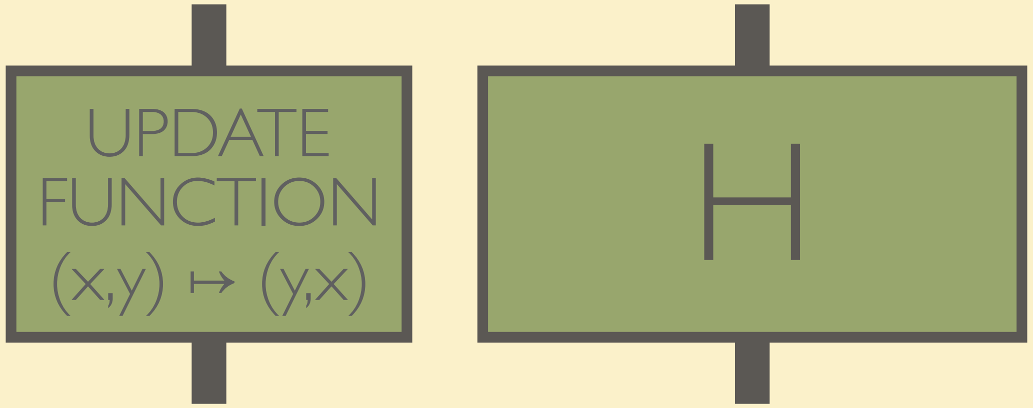

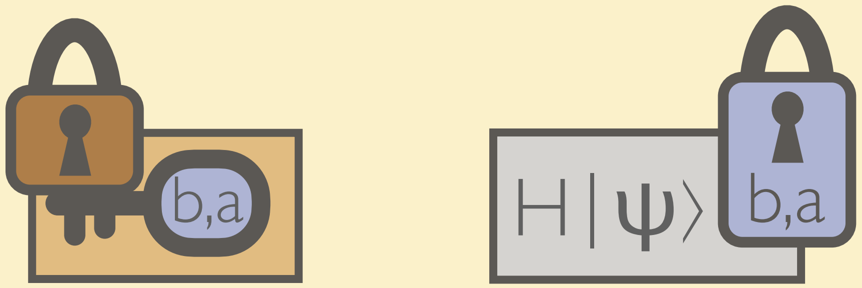

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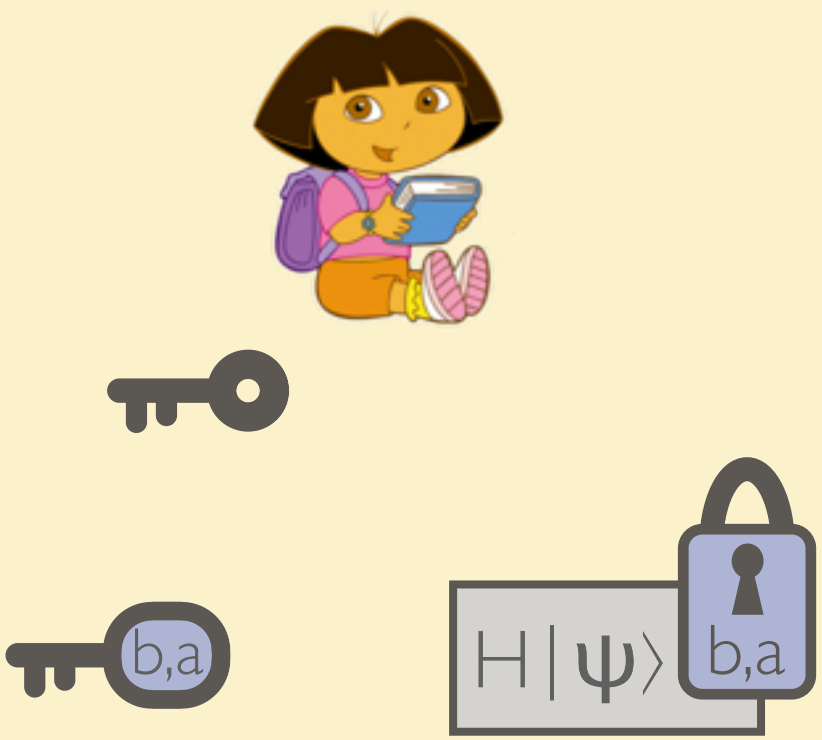

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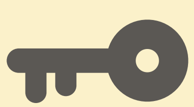

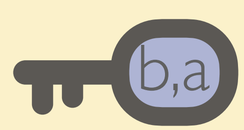

$H|\psi\rangle$

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THE CHALLENGE: T GATE

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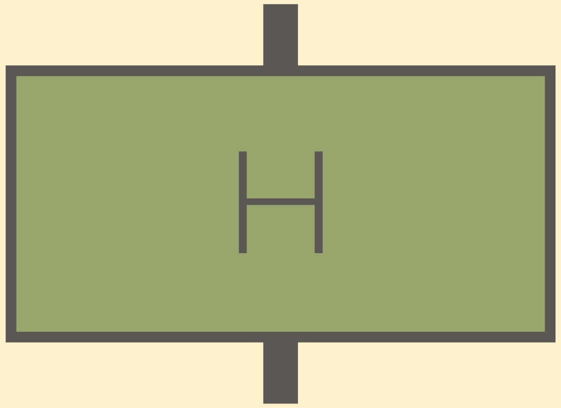

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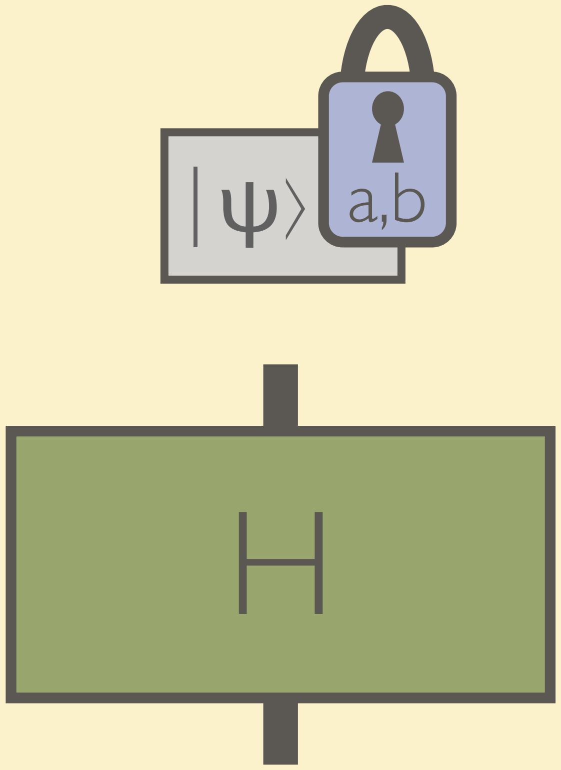

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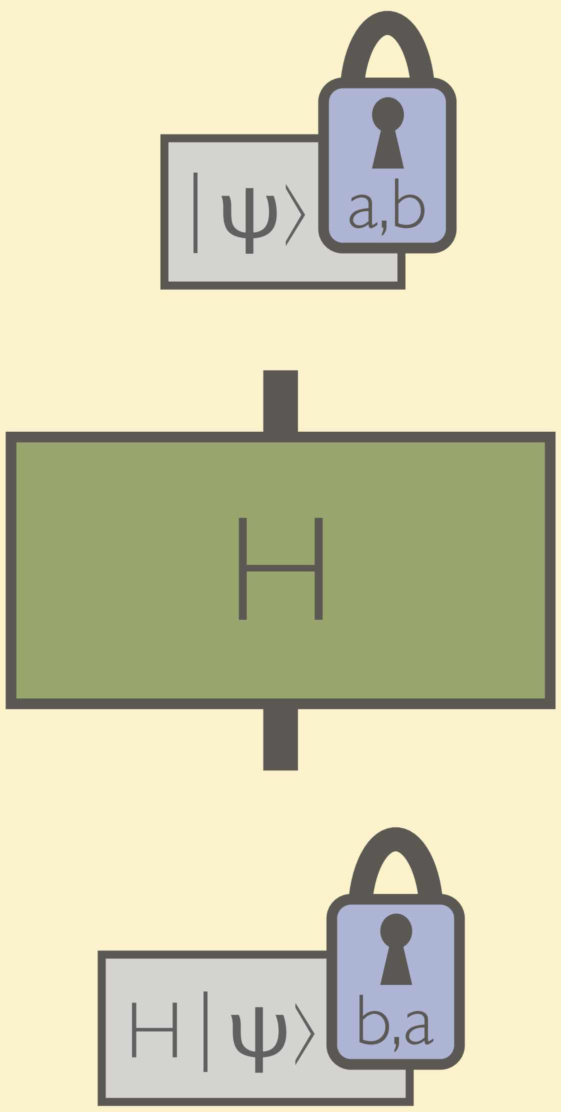

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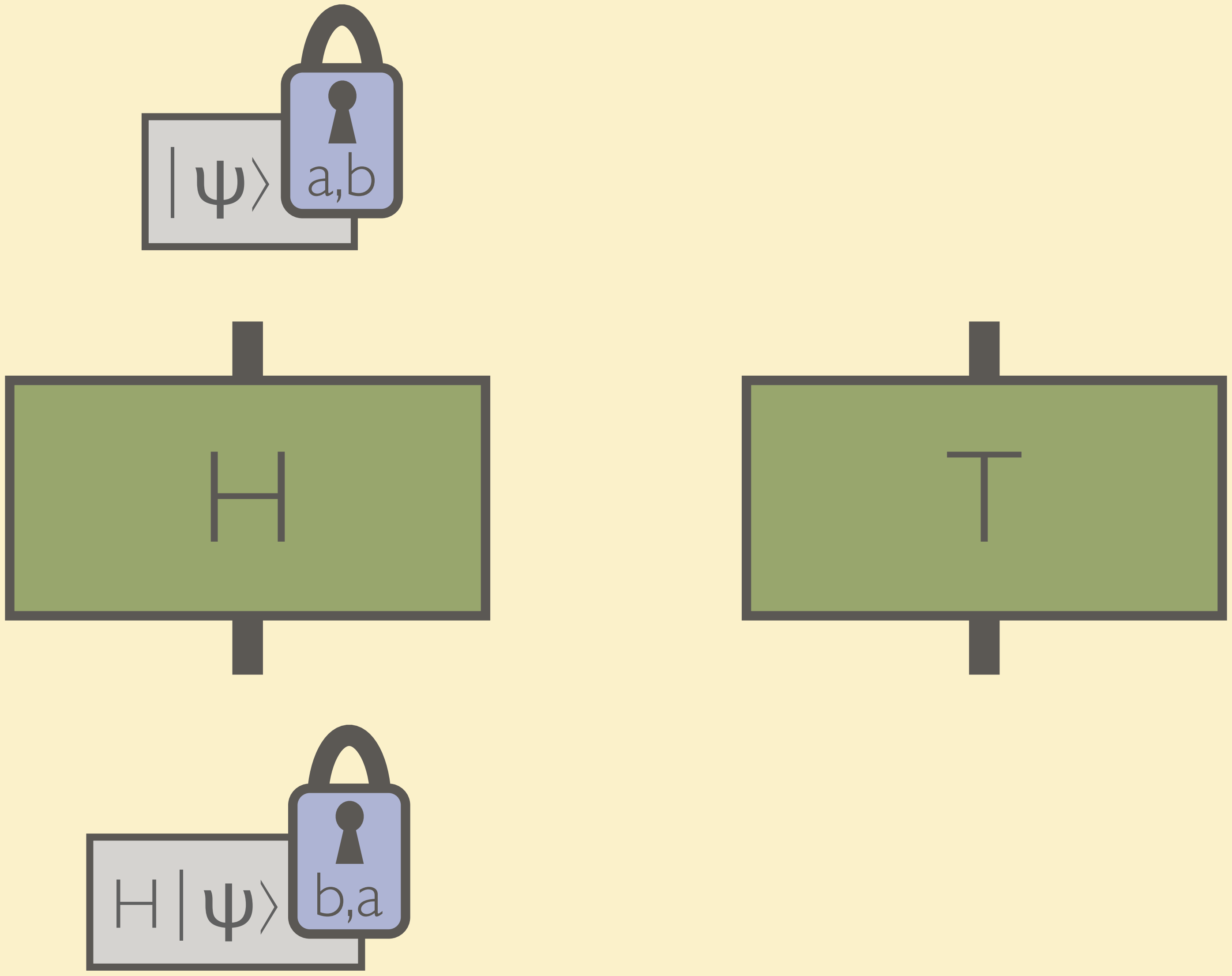

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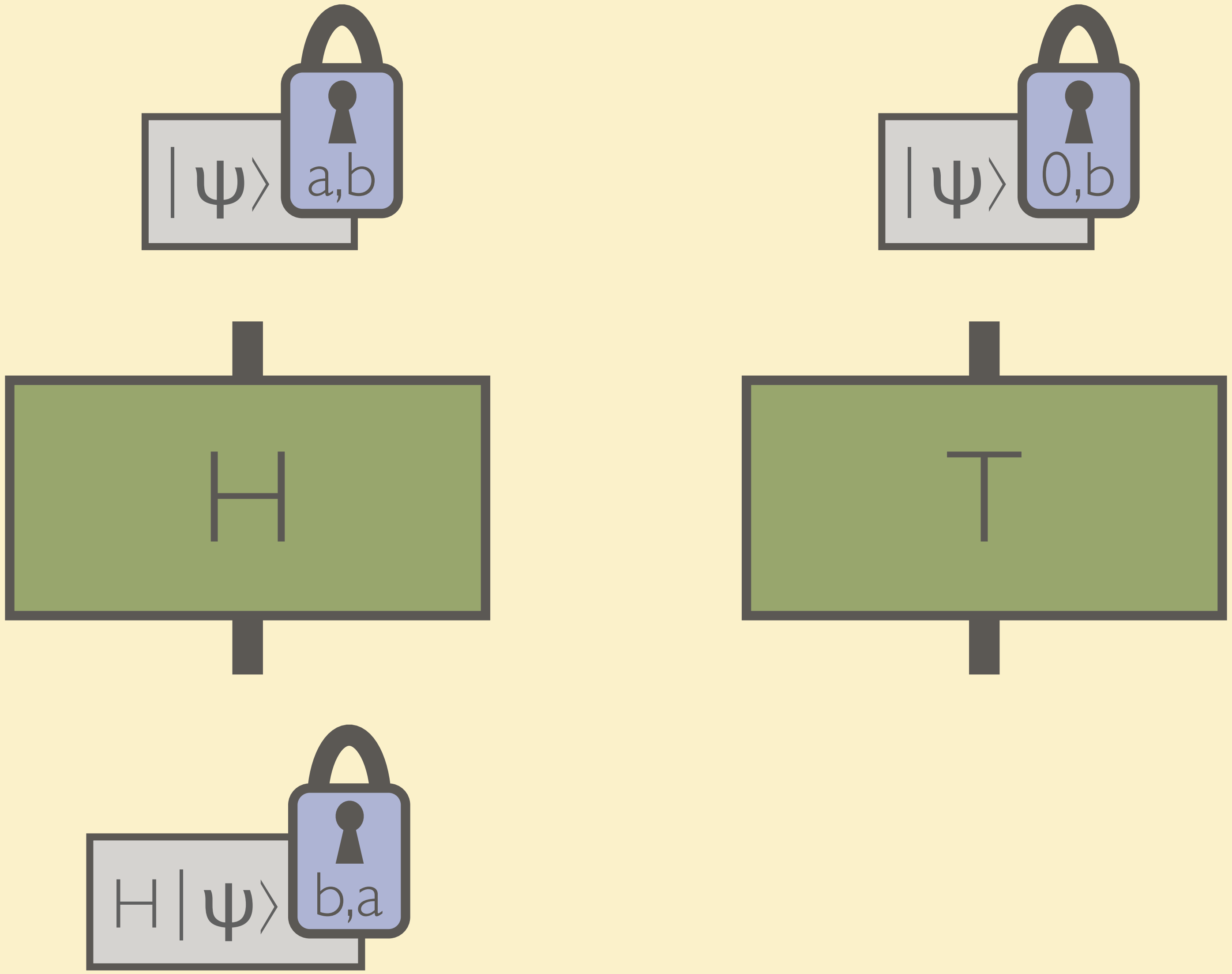

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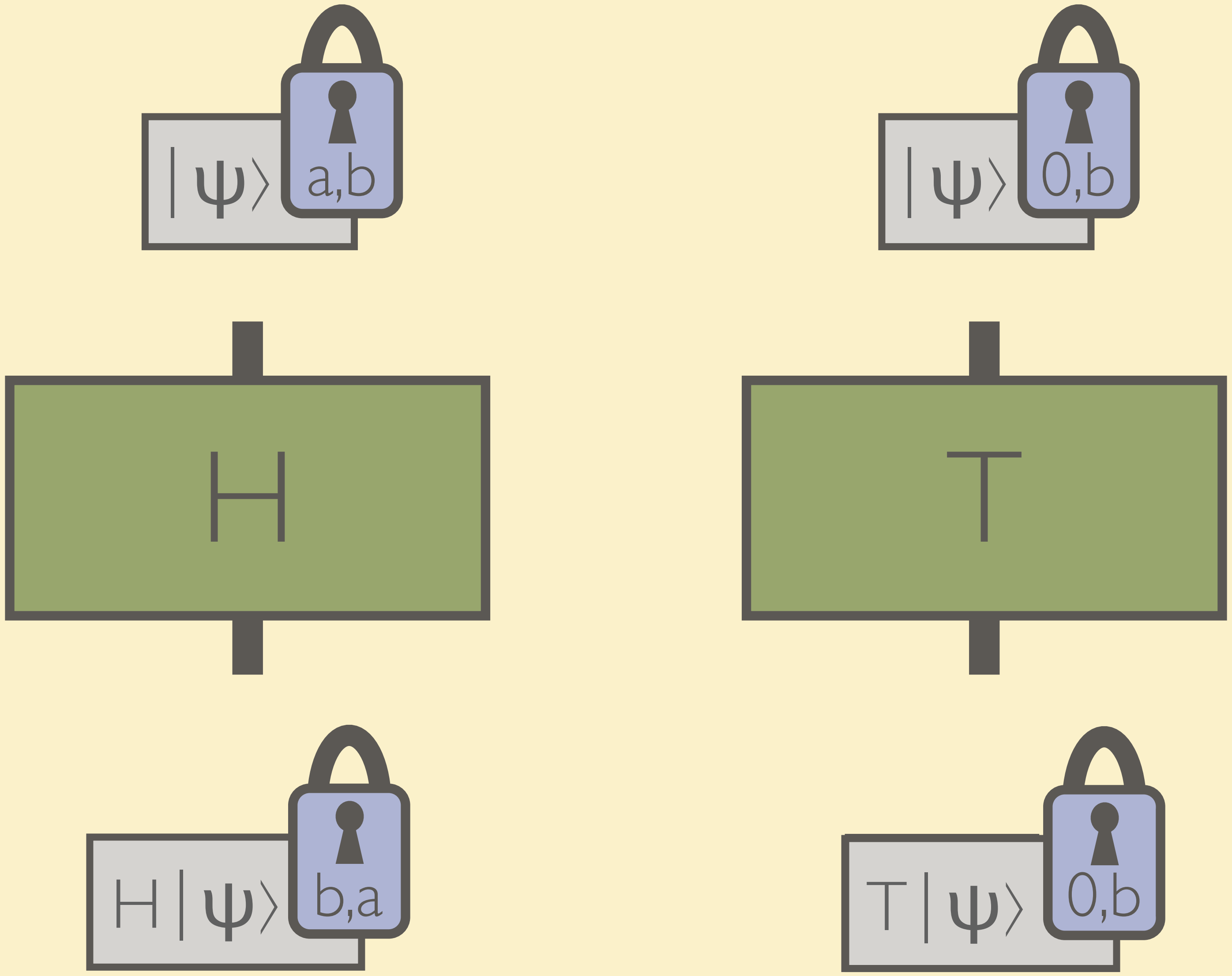

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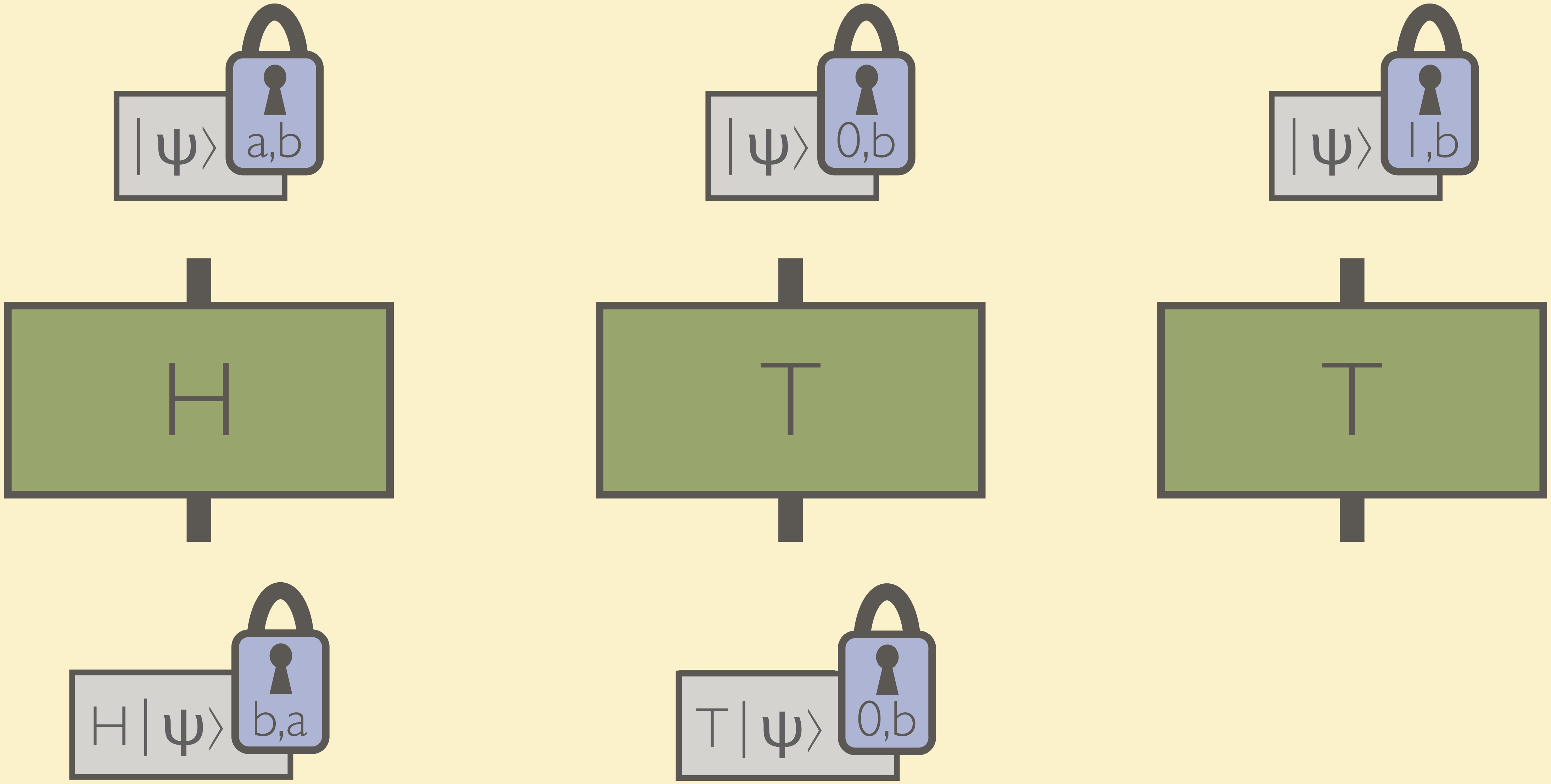

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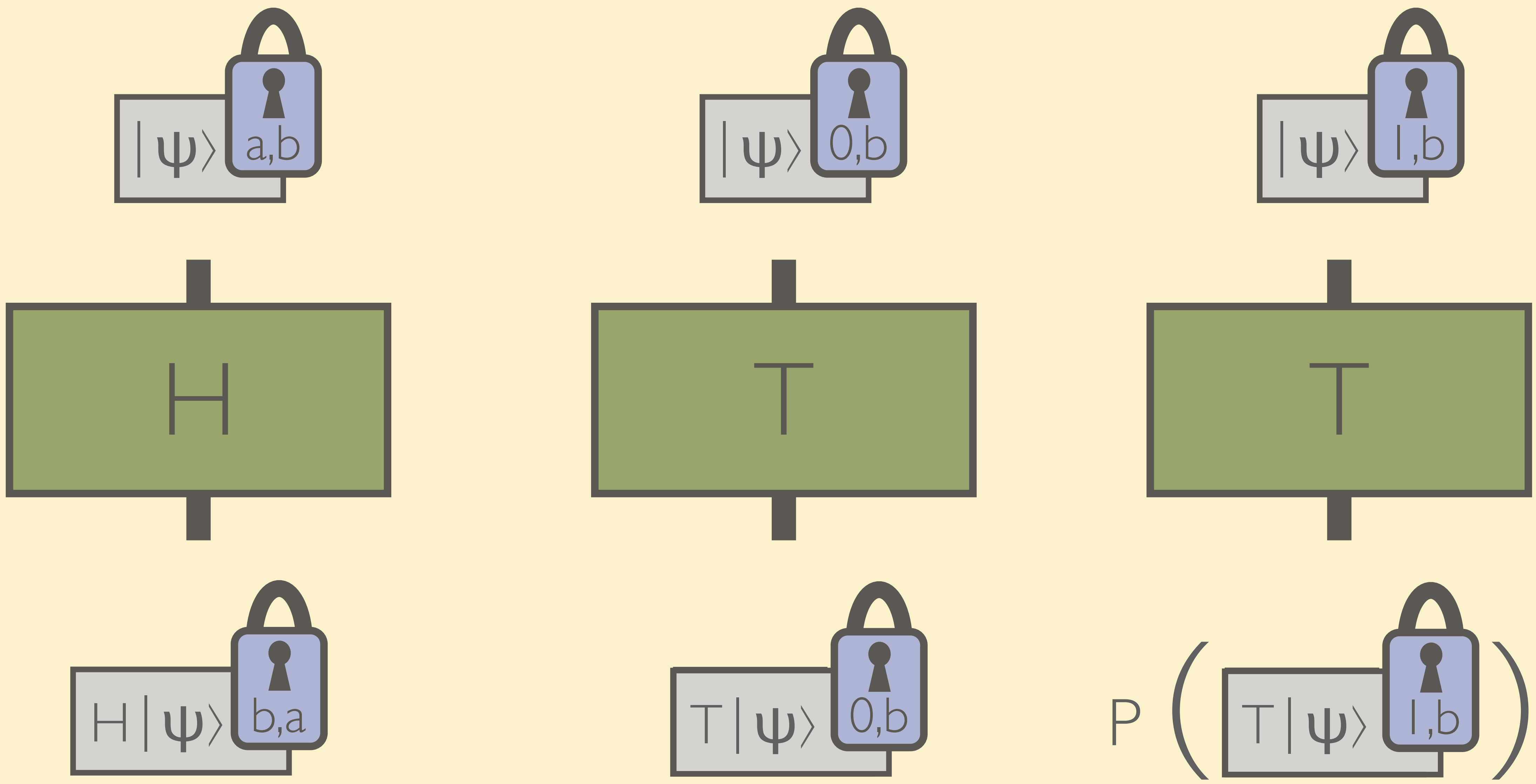

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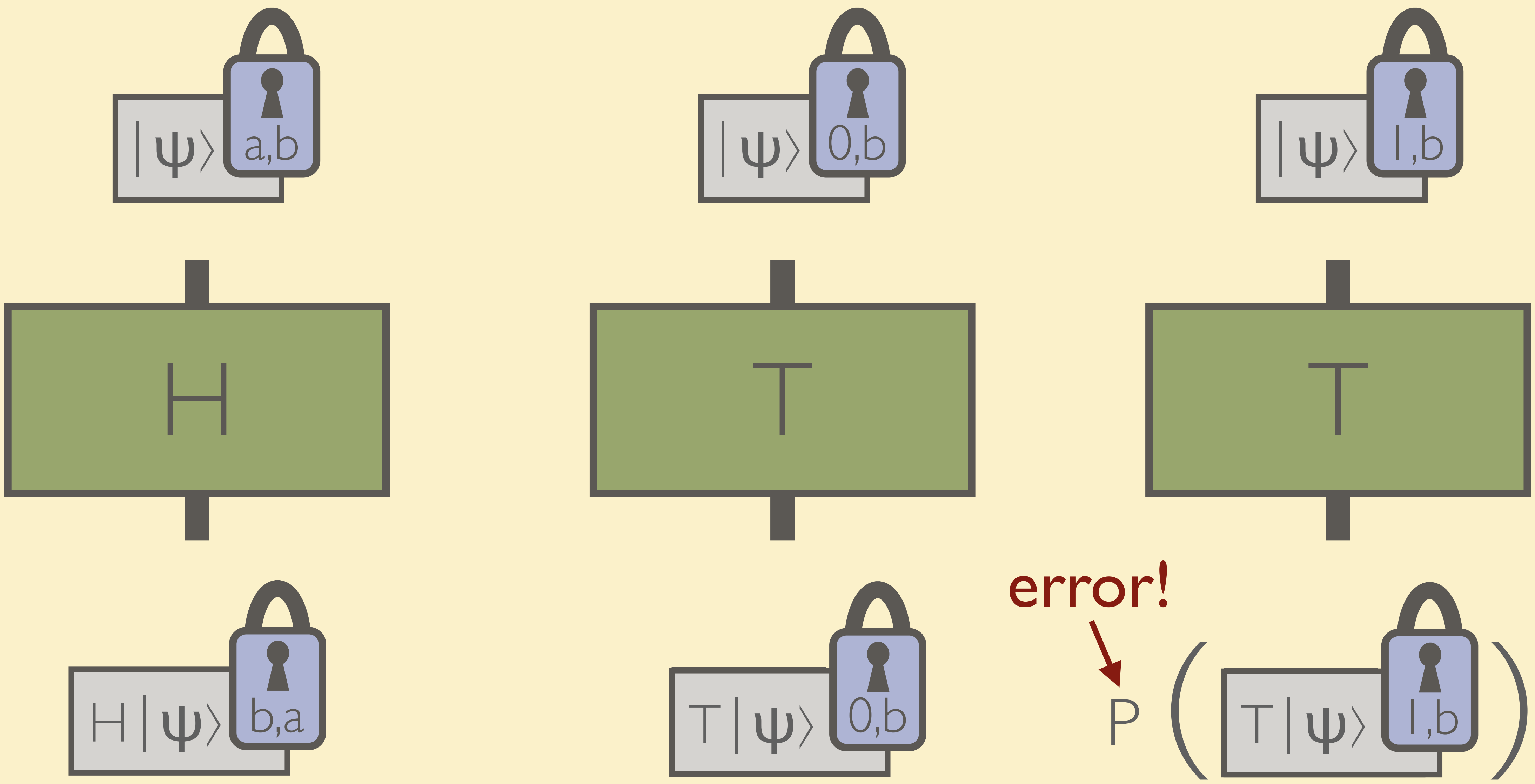

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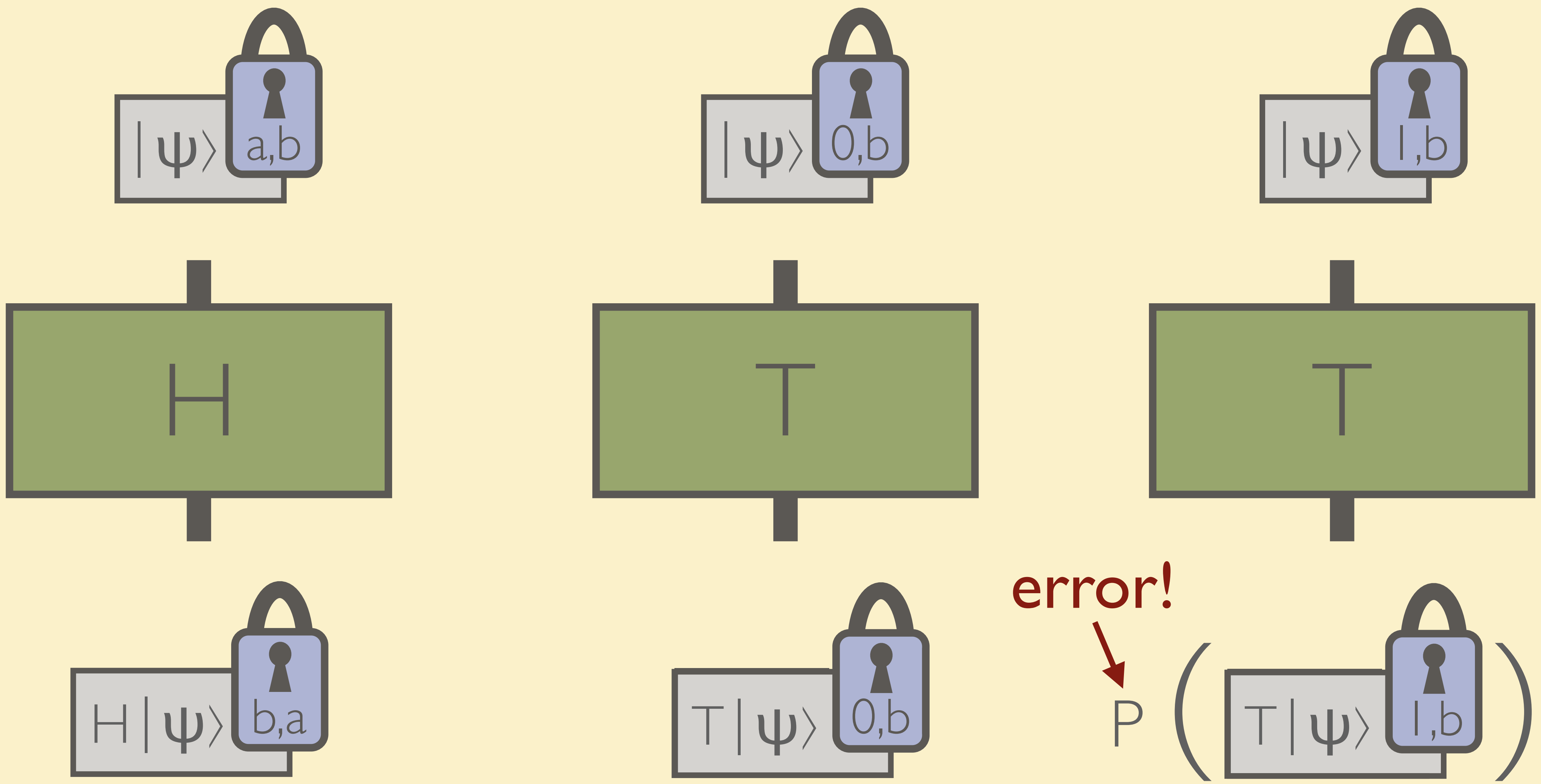

how to apply correction P^{-1} iff $a = 1$?

PREVIOUS RESULTS: OVERVIEW

(comparison based on Stacey Jeffery's slides)

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[OTF15] Y. Ouyang, S-H. Tan, J. Fitzsimons. Quantum homomorphic encryption from quantum codes. arxiv:1508.00938

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	homomorphic for	compactness	security
Not encrypting	Quantum circuits	yes	no
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Quantum OTP	no	yes	inf theoretic
Clifford Scheme	Clifford circuits	yes	computational

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Our result	QCircuits of polynomial size (levelled FHE)	yes	computational

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- ✓ HOMOMORPHIC ENCRYPTION
- ✓ PREVIOUS RESULTS
3. NEW RESULT

ERROR-CORRECTION “GADGET”

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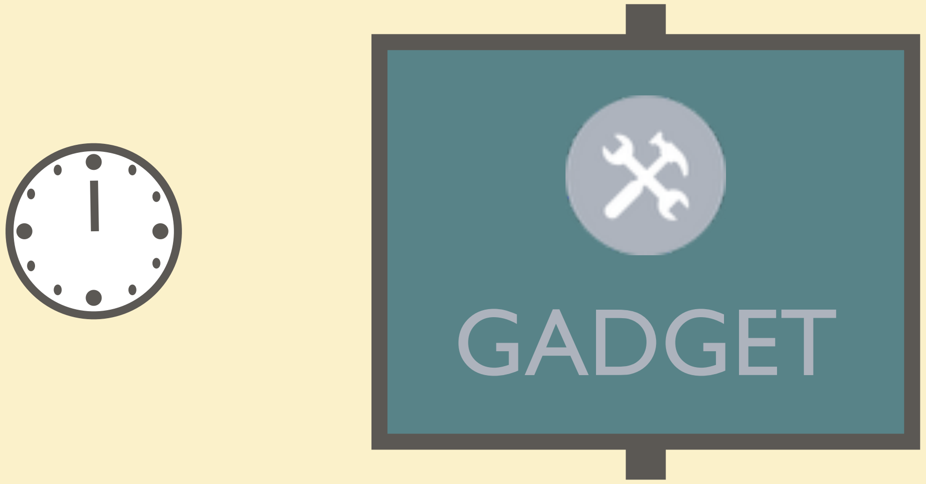
A quantum state that:

- can be efficiently constructed and used



ERROR-CORRECTION "GADGET"

A quantum state that:

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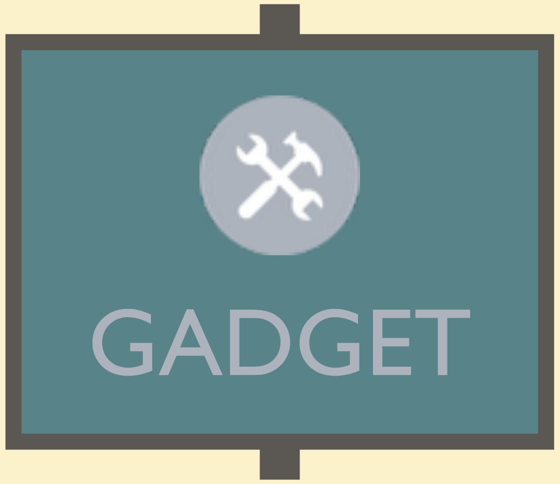
- applies correction iff error was present (iff a = 1)



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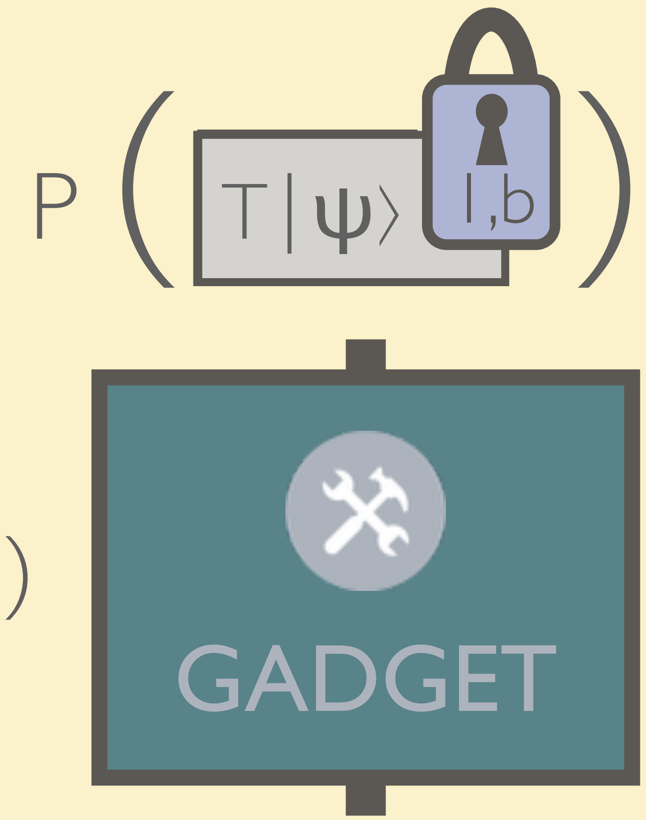

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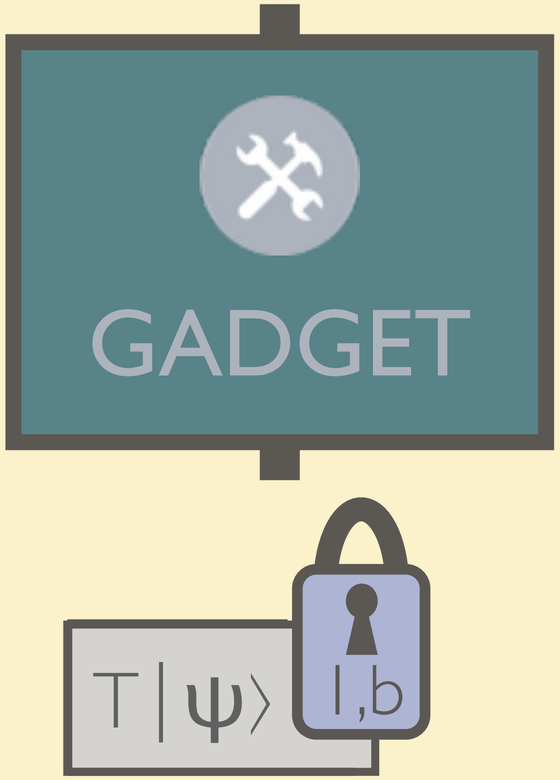

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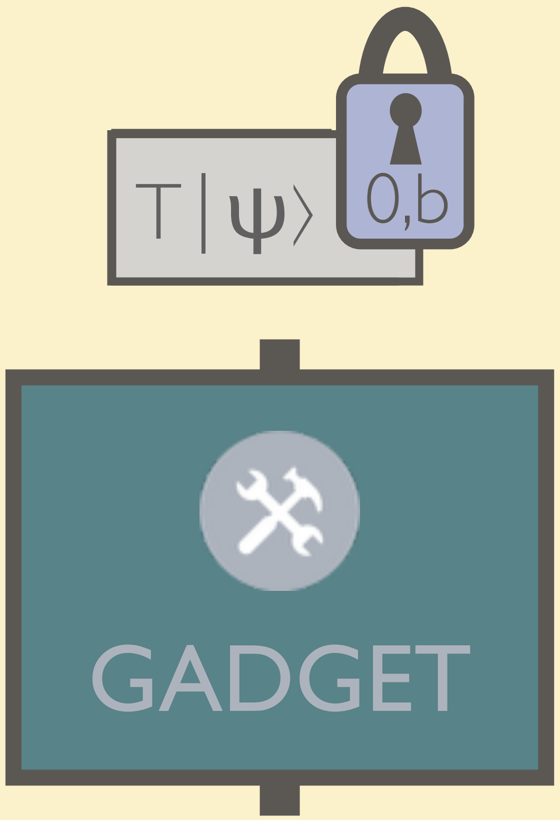

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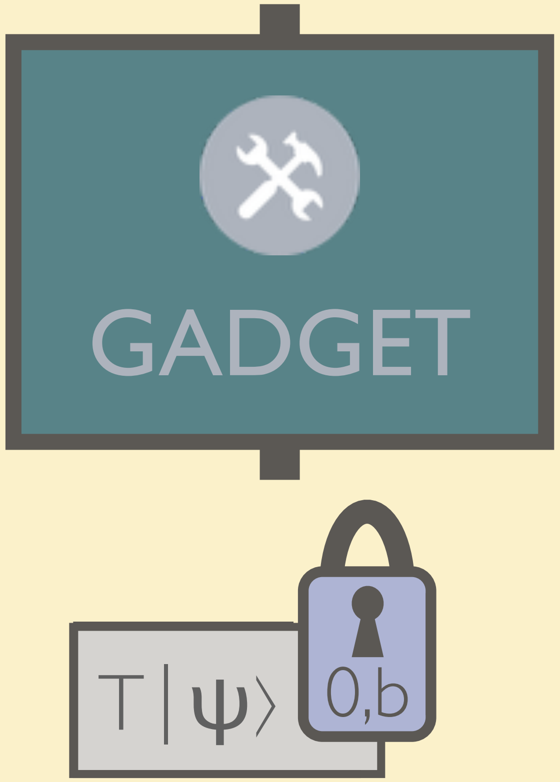
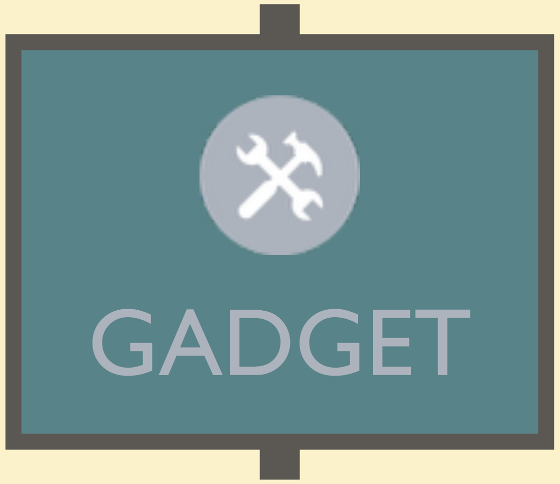


ERROR-CORRECTION "GADGET"

A quantum state that:

- can be efficiently constructed and used
- applies correction iff error was present (iff a = 1)
- is destroyed after a single use



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EXCURSION

Theoretical Computer Science

PERMUTATION BRANCHING PROGRAM

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- computes some Boolean function $f(x,y)$

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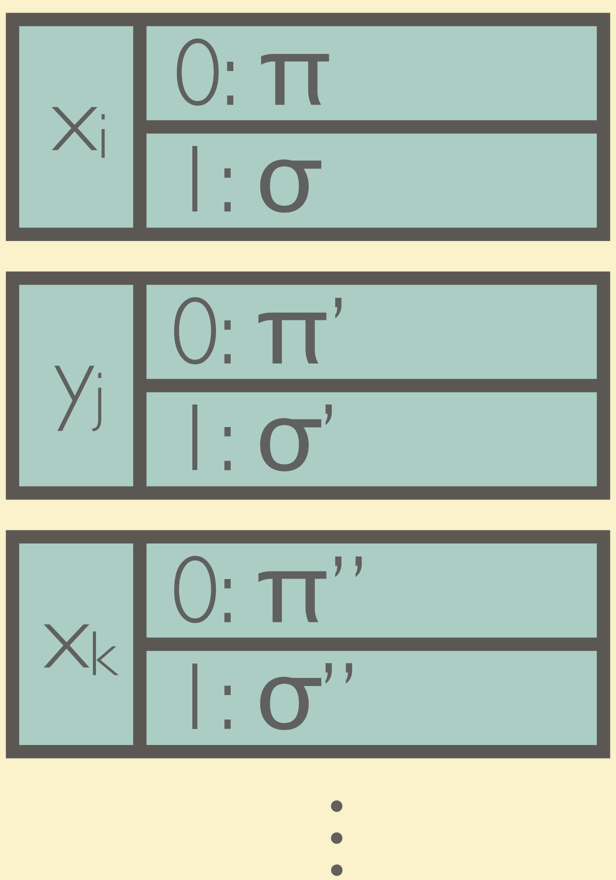

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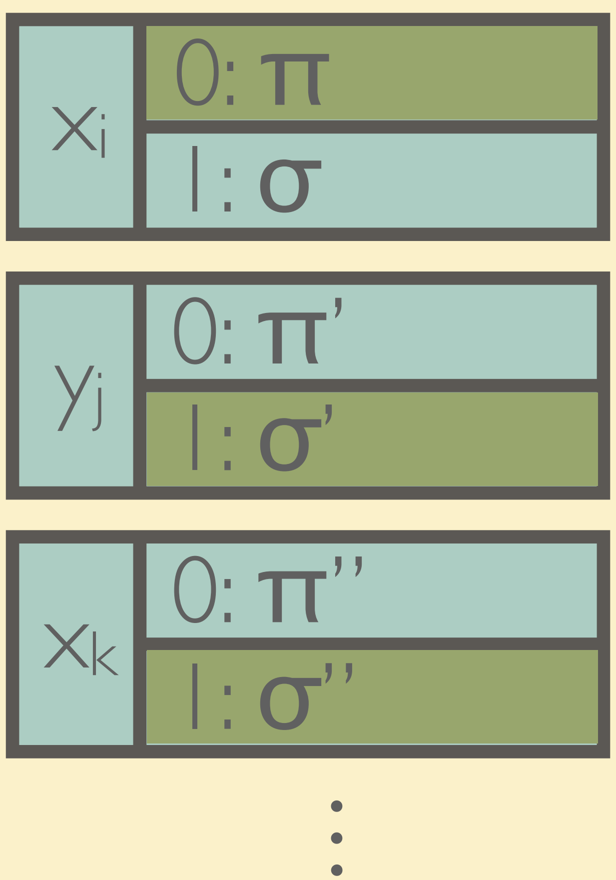
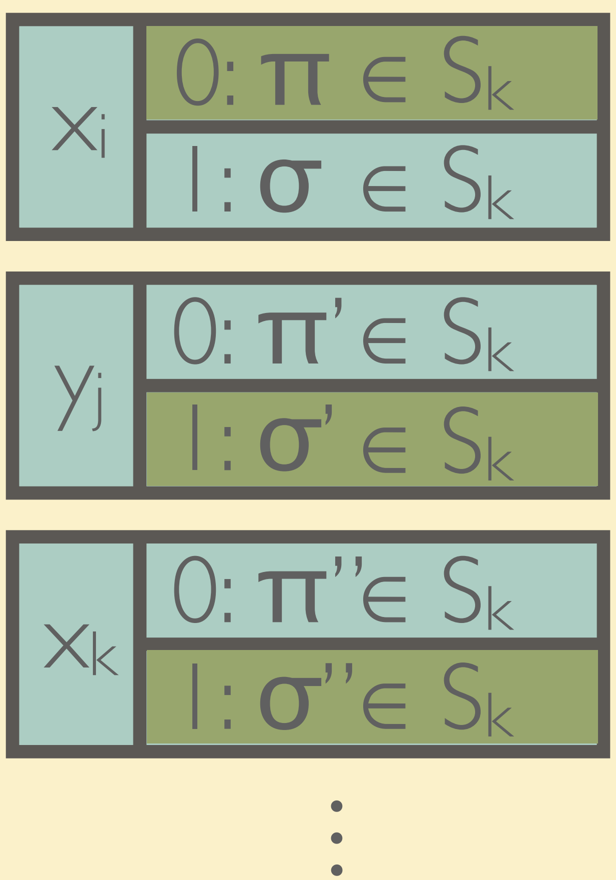


PERMUTATION BRANCHING PROGRAM

- computes some Boolean function f(x,y)
- list of instructions: permutations of {1,2, …, k}



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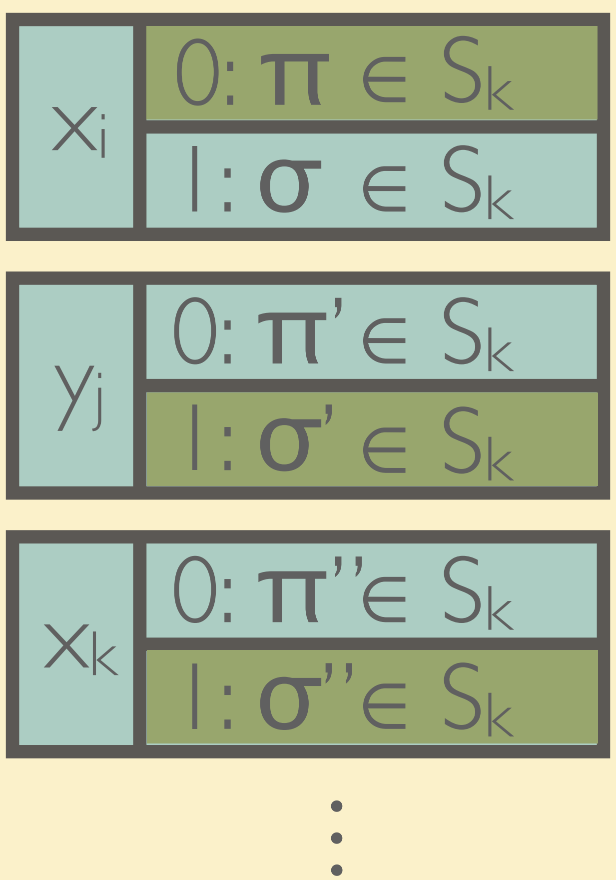

output: $\ldots \circ \sigma'' \circ \sigma' \circ \pi$

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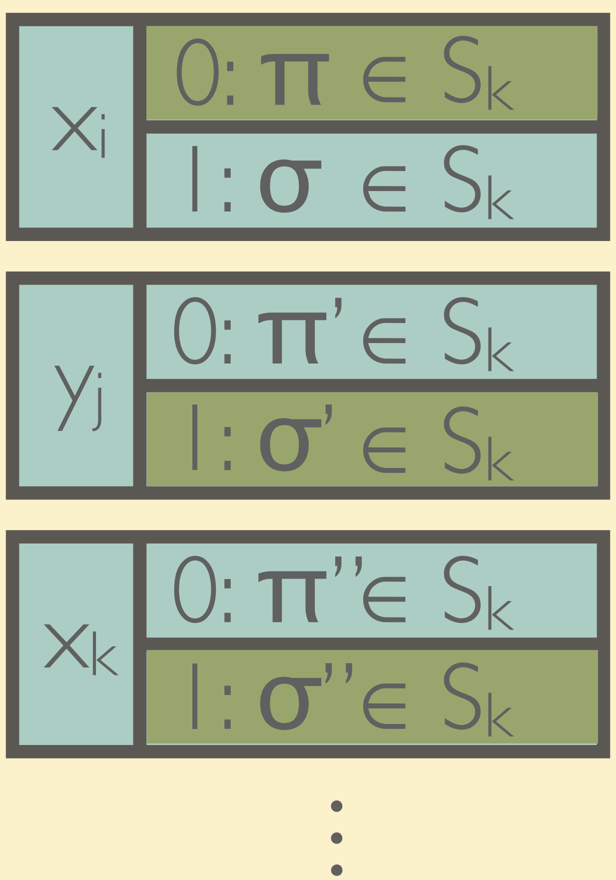

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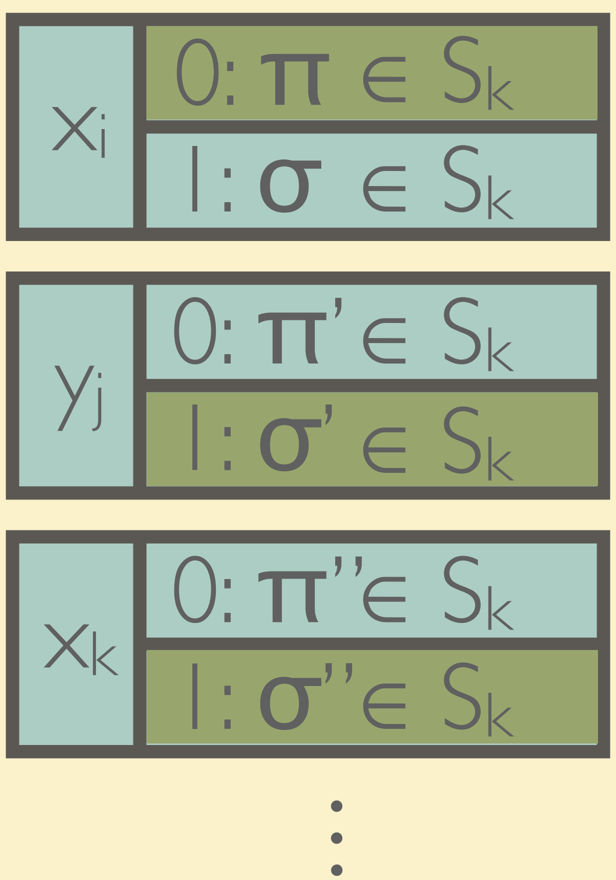

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- (fixed) cycle

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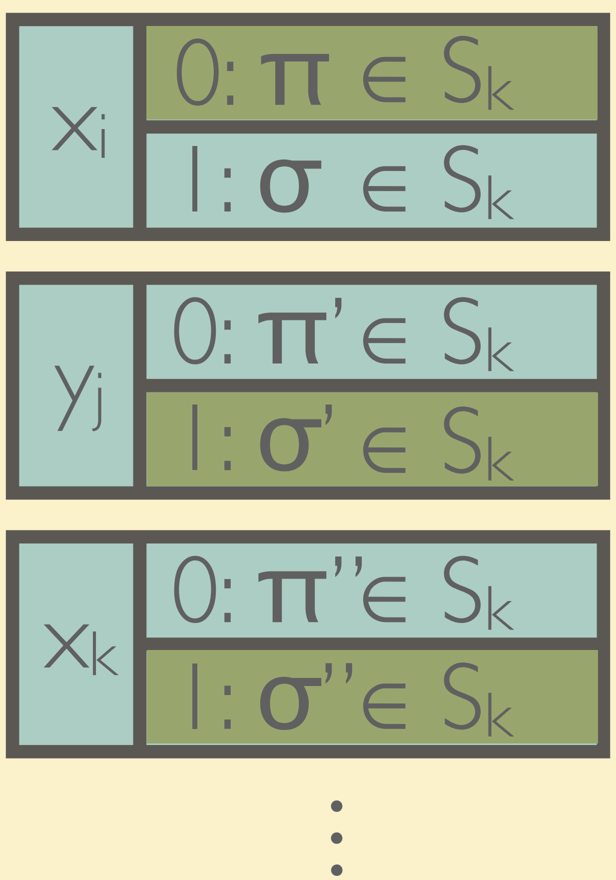

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- id $\Rightarrow f(x,y) = 0$
- (fixed) cycle

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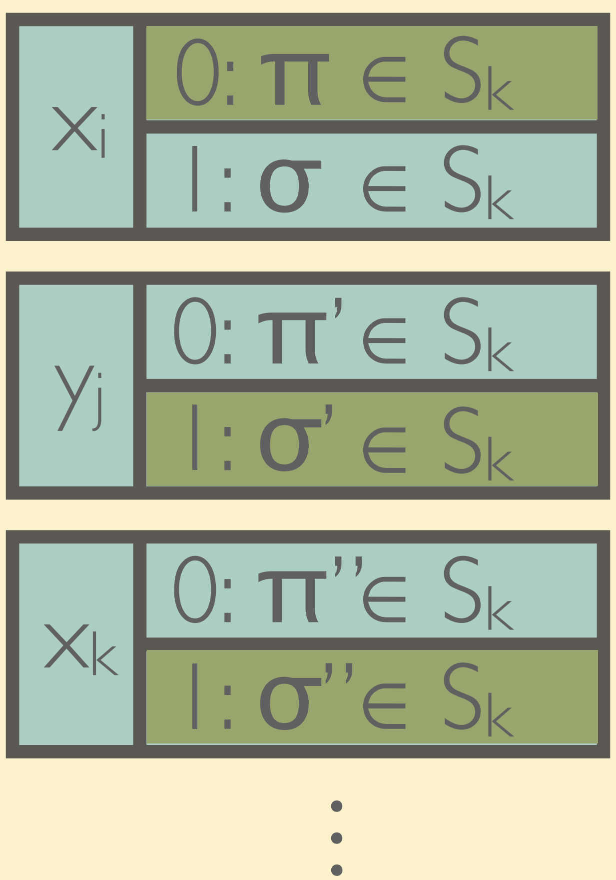

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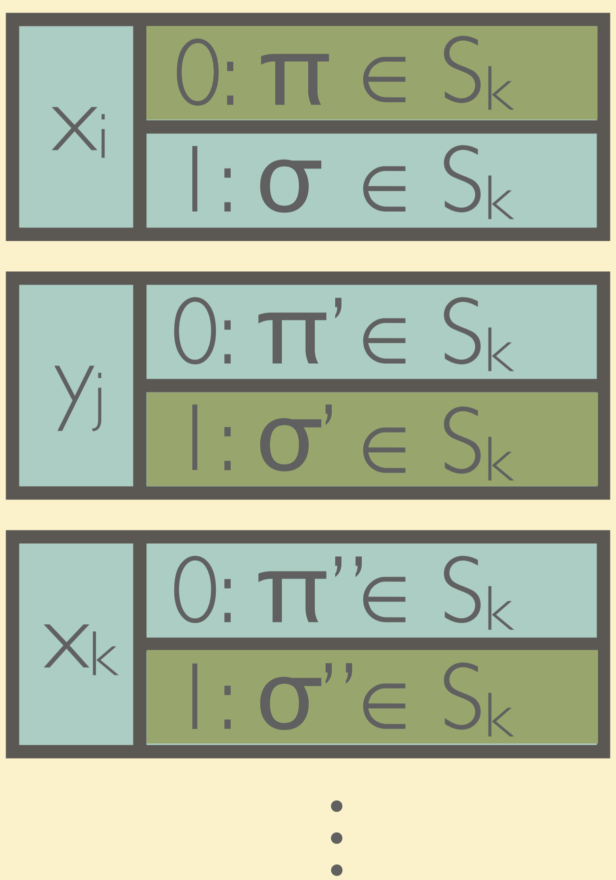

output: … ∘ **σ**’’ ∘ **σ**’ ∘ **π**

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length: # of instructions

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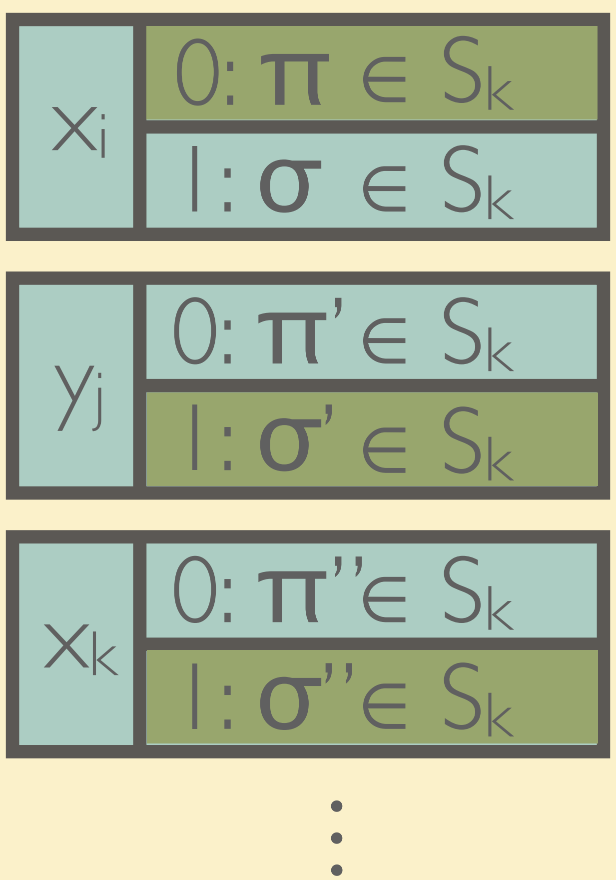

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width: k

EXAMPLE PBP (OR)

length 4, width 5:

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x_1	0: (12345)	1: id
y_1	0: (12453)	1: id
x_1	0: (54321)	1: id
y_1	0: (15243)	1: (14235)

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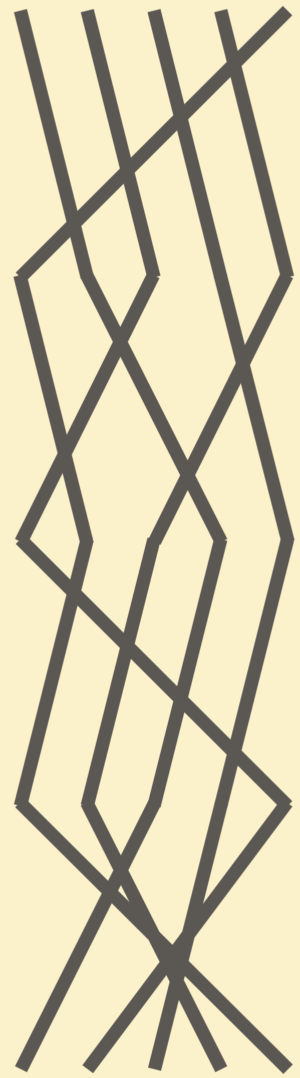
OR(0,0)



output: id

0

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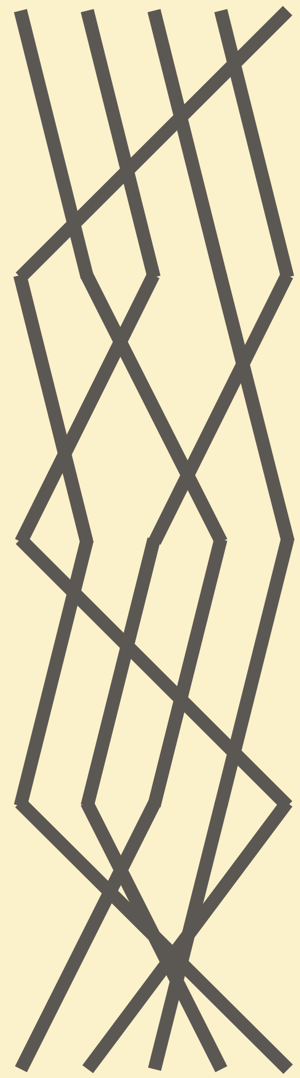
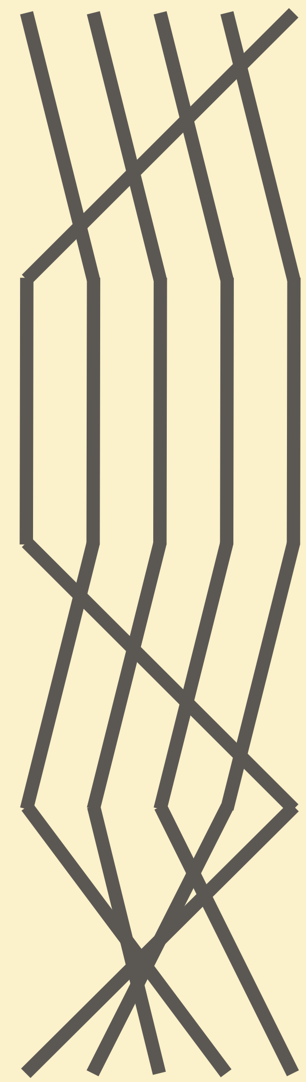


OR(0,1)



	OR(0,0)	OR(0,1)
output:	id	(14235)
	0	1

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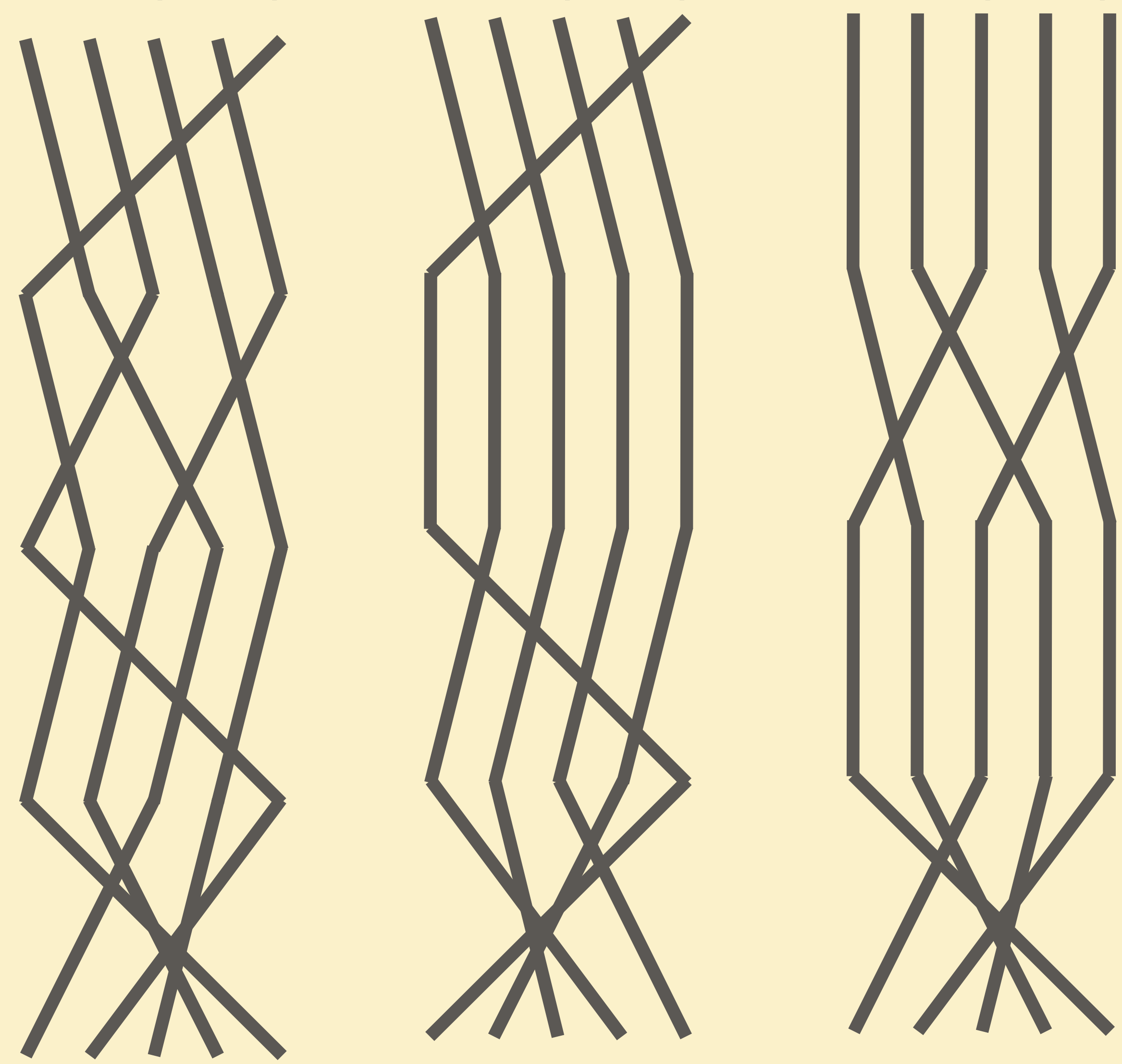

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OR(0,0)	OR(0,1)	OR(1,0)	OR(1,1)
id	(14235)	(14235)	
0	1	1	

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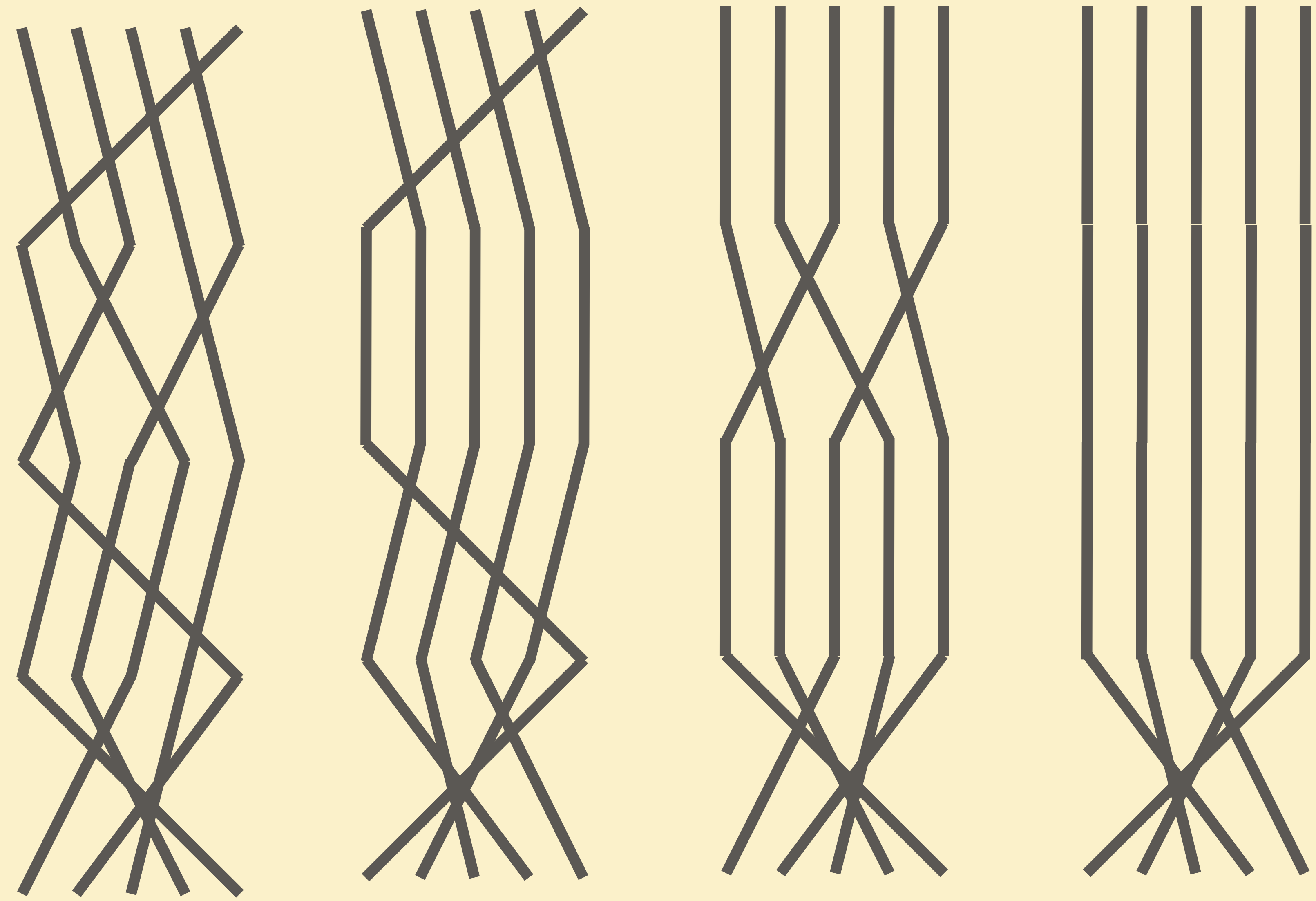

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BARRINGTON'S THEOREM

Theorem (variation): if $f : \{0,1\}^n \times \{0,1\}^m \rightarrow \{0,1\}$ is in NC^1,
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BARRINGTON'S THEOREM

Theorem (variation): if $f : \{0,1\}^n \times \{0,1\}^m \rightarrow \{0,1\}$ is in NC^1, then there exists a permutation branching program for f with:

- width 5
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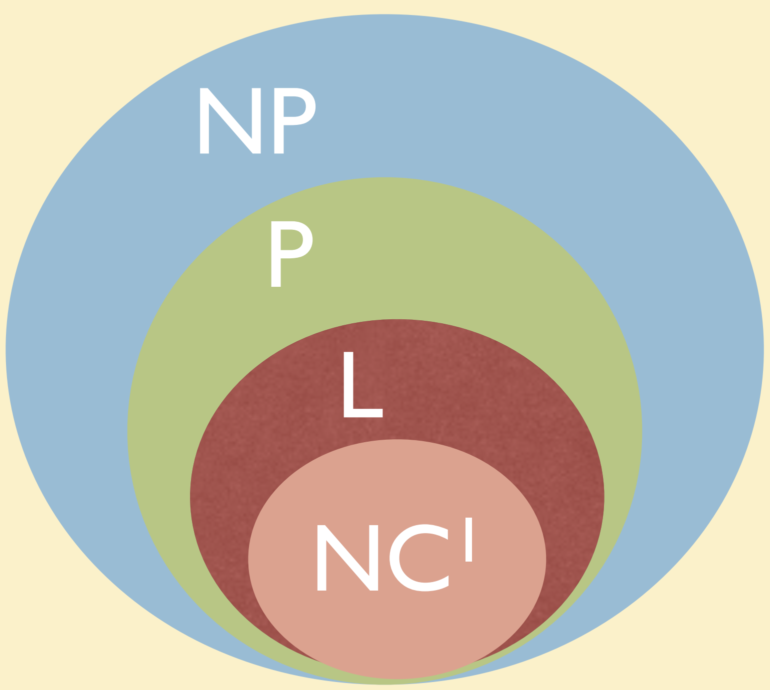

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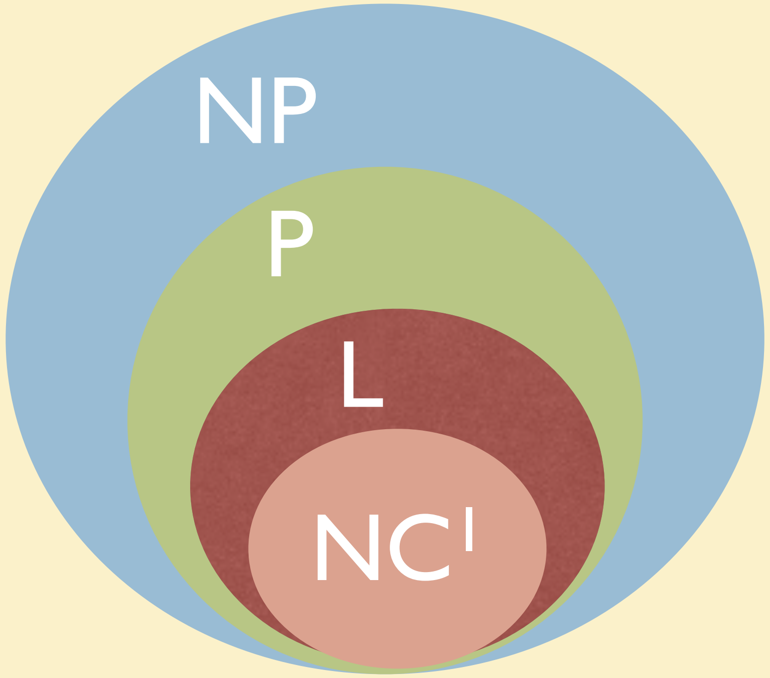

Classical homomorphic decryption functions happen to be in NC^1… [BV11]

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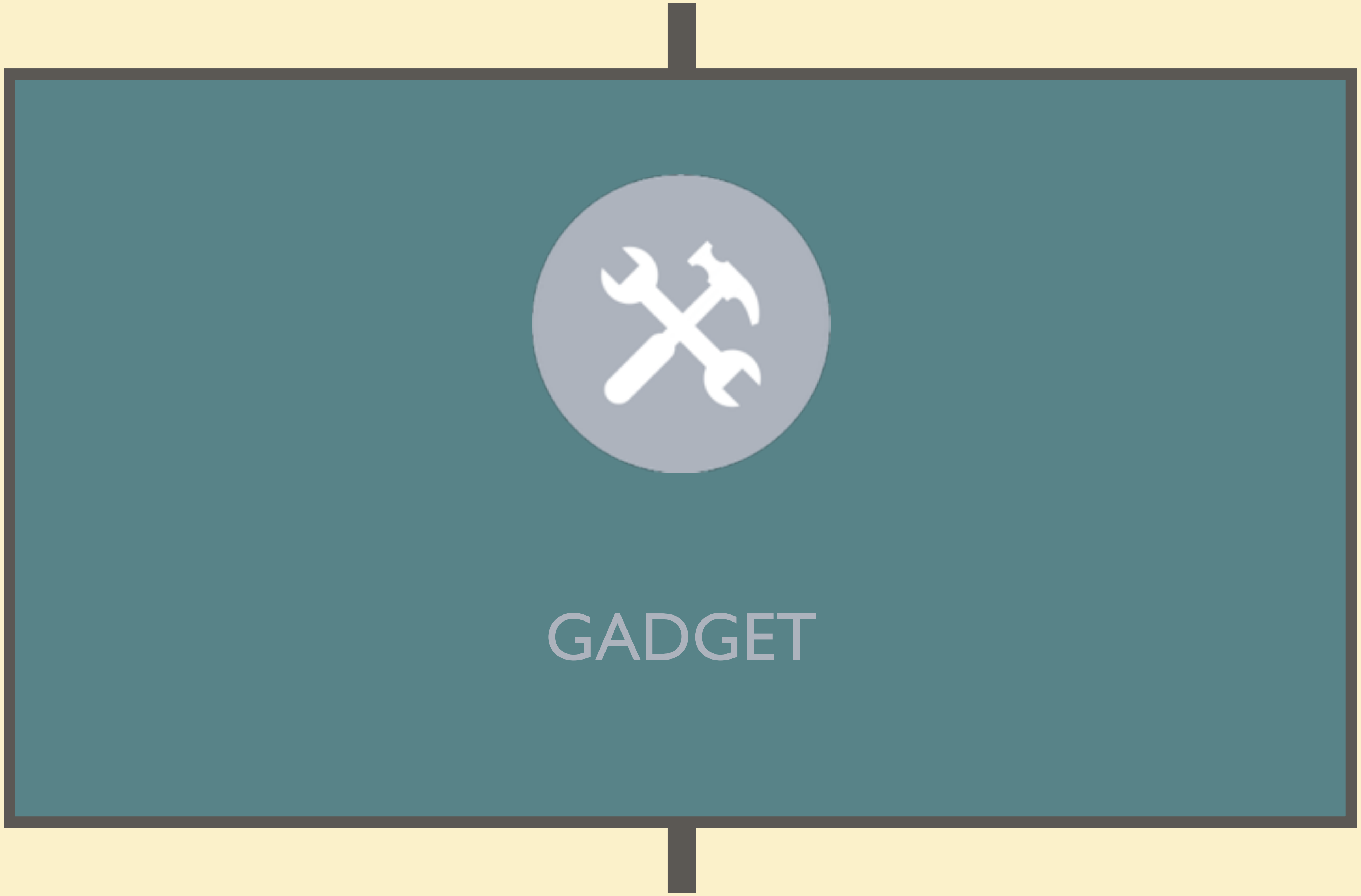

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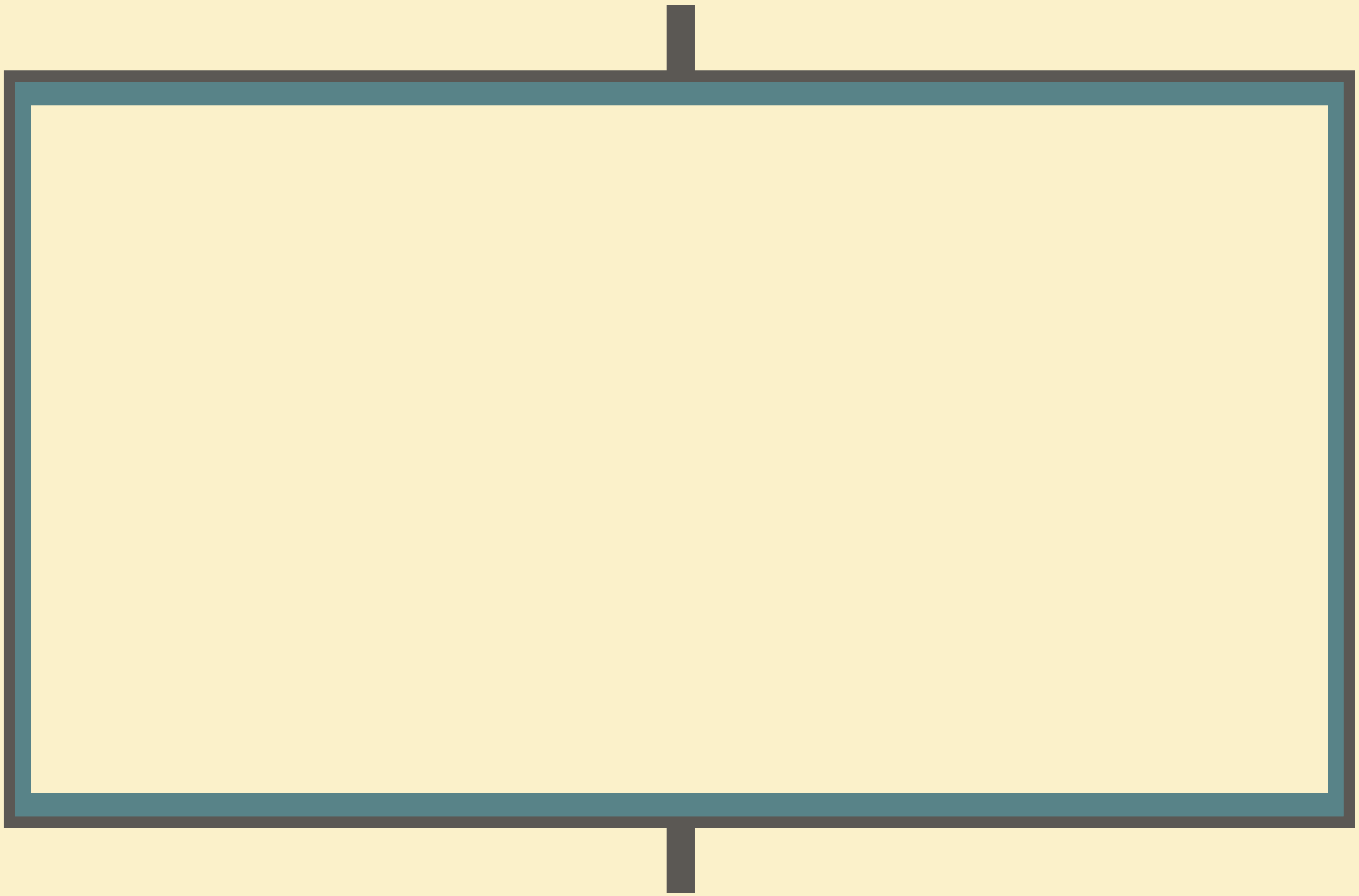

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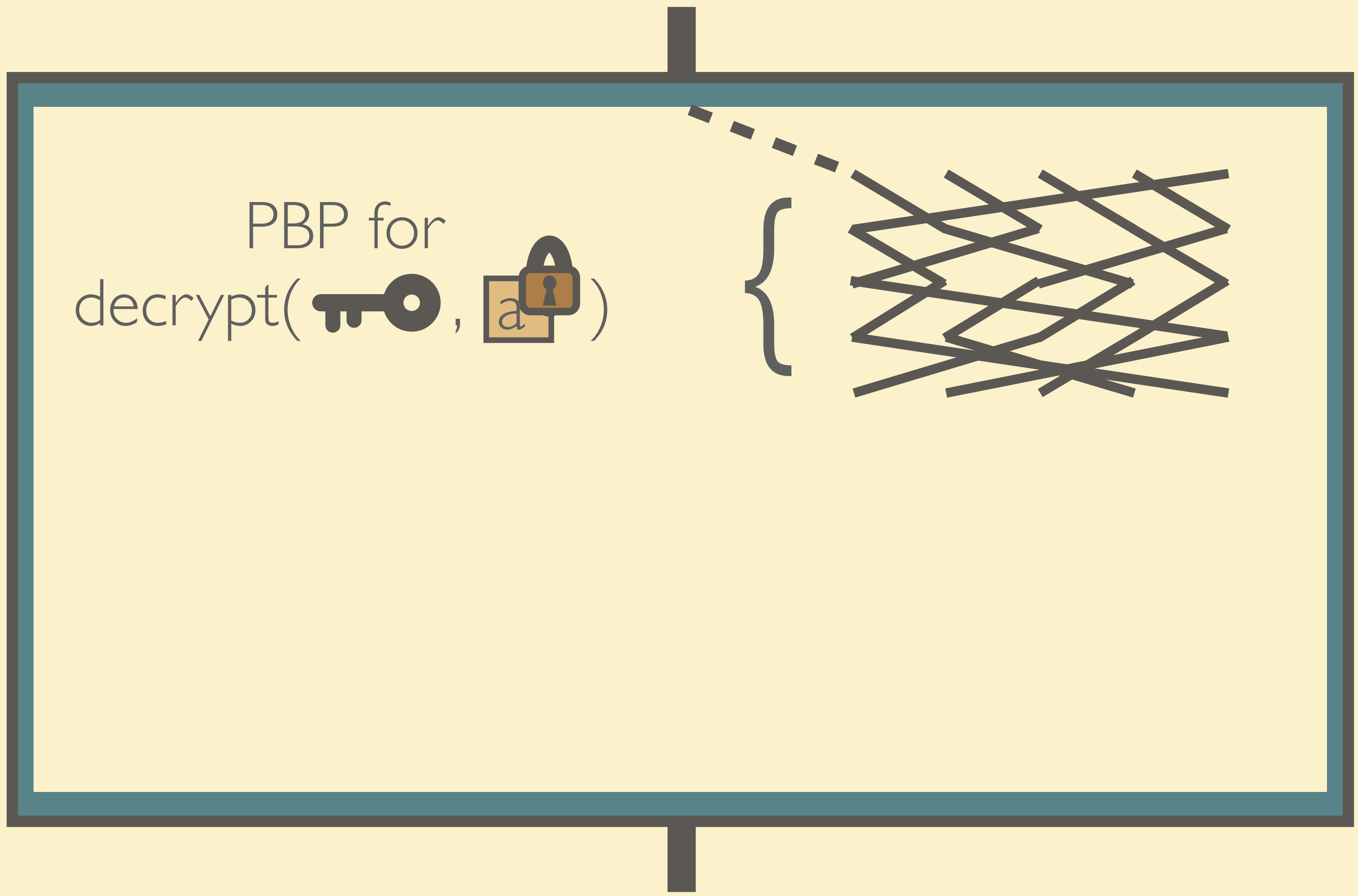

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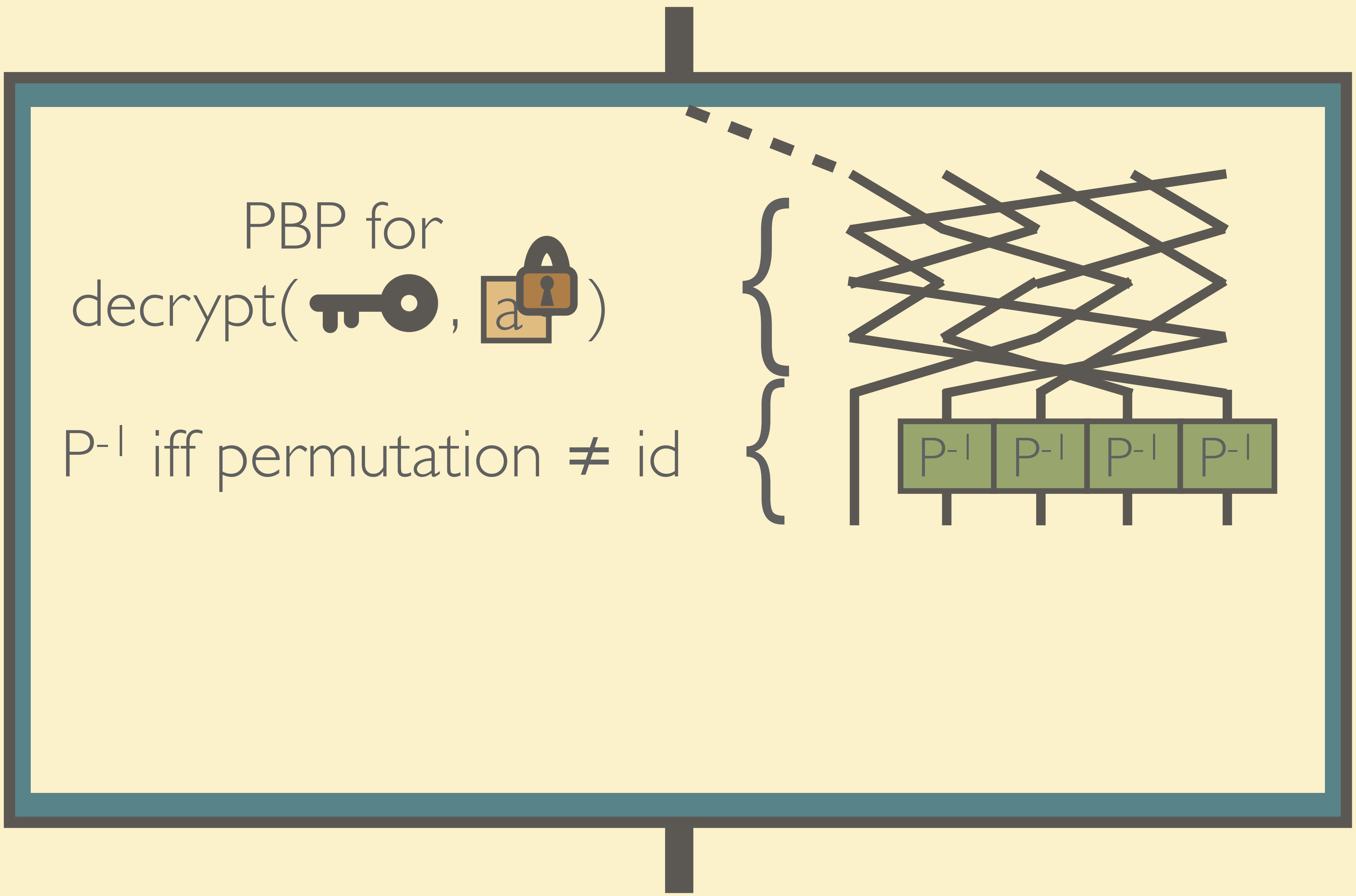

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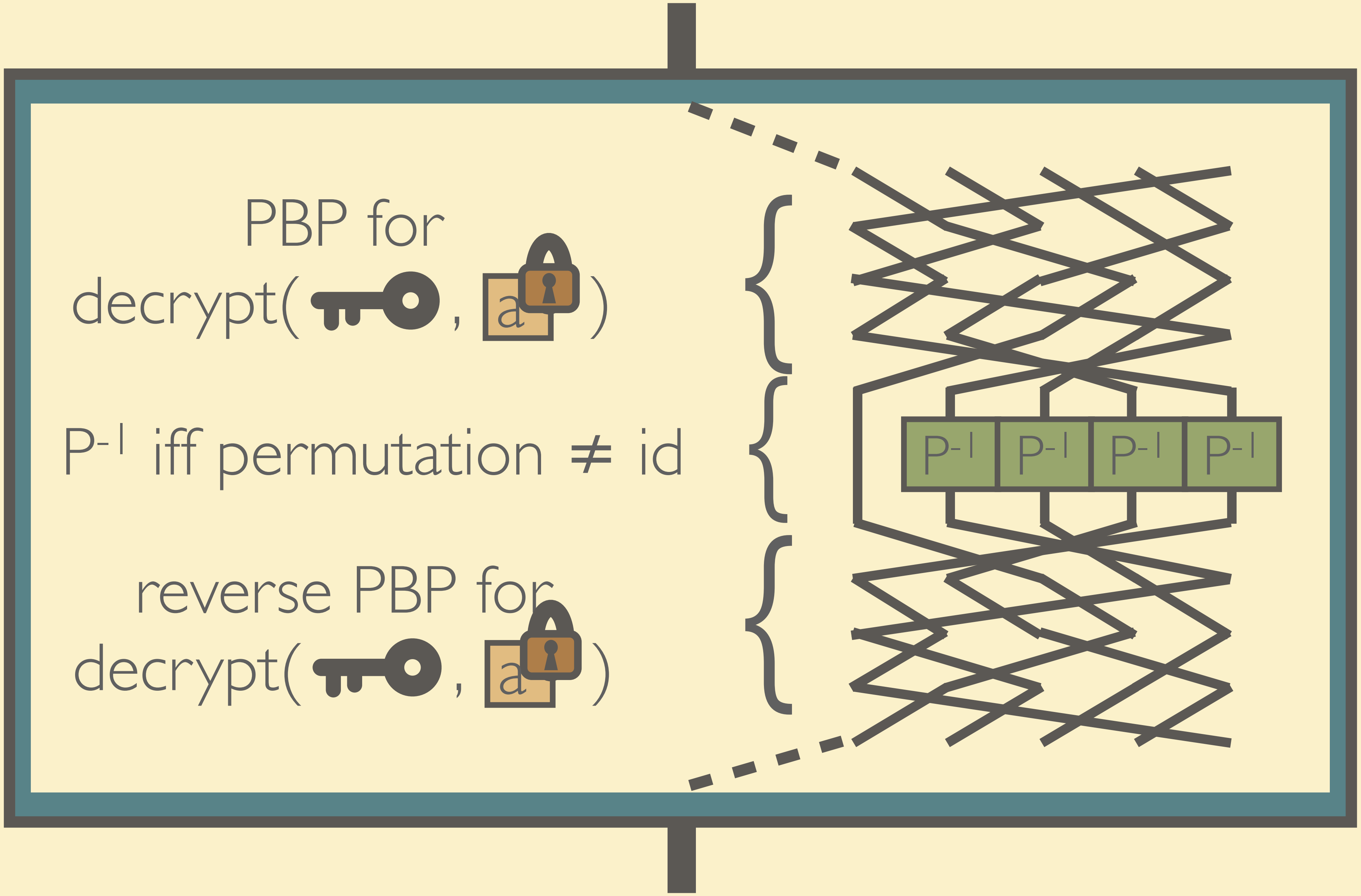

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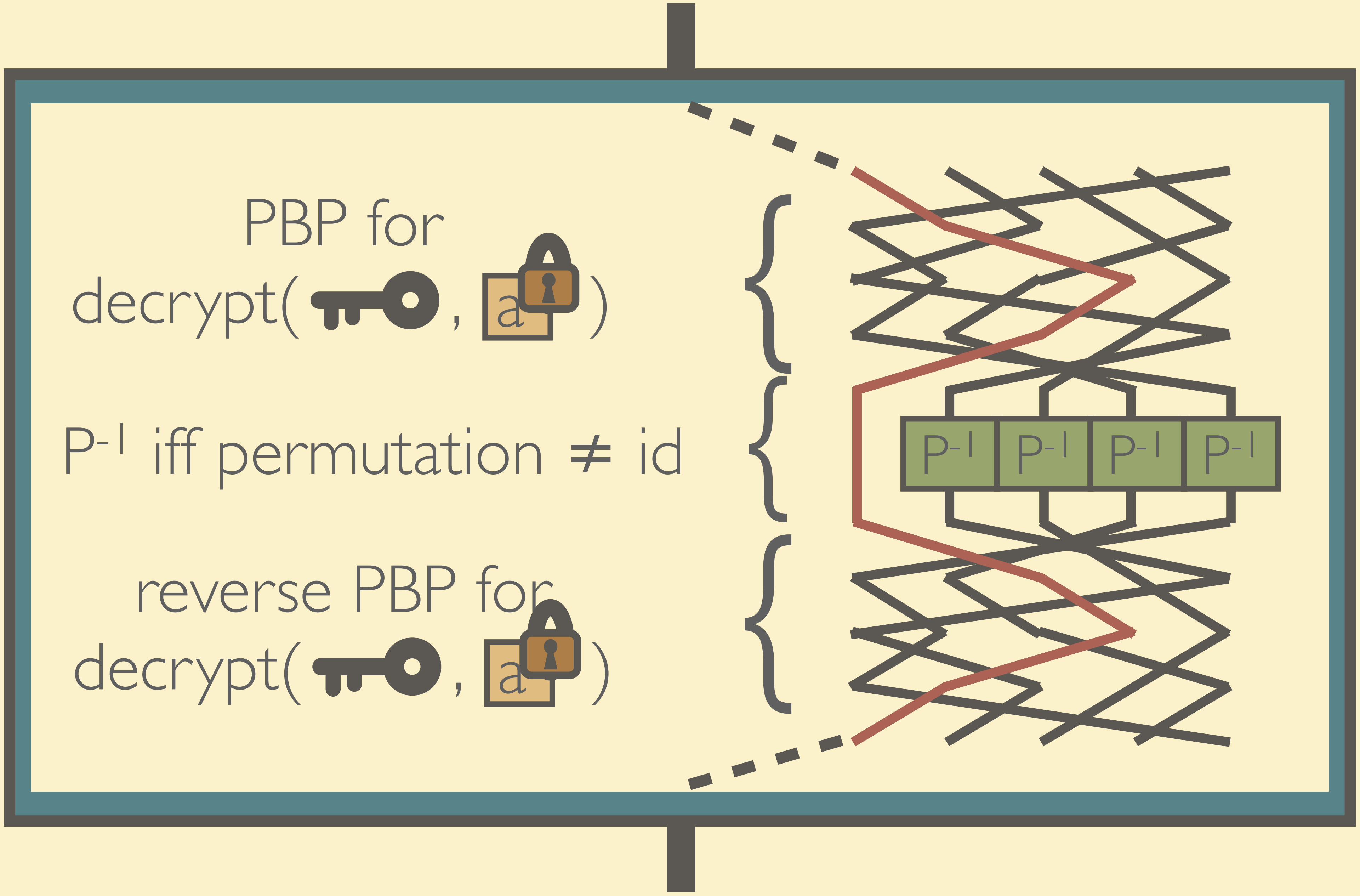

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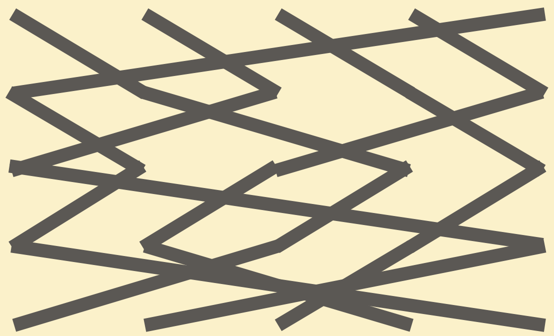

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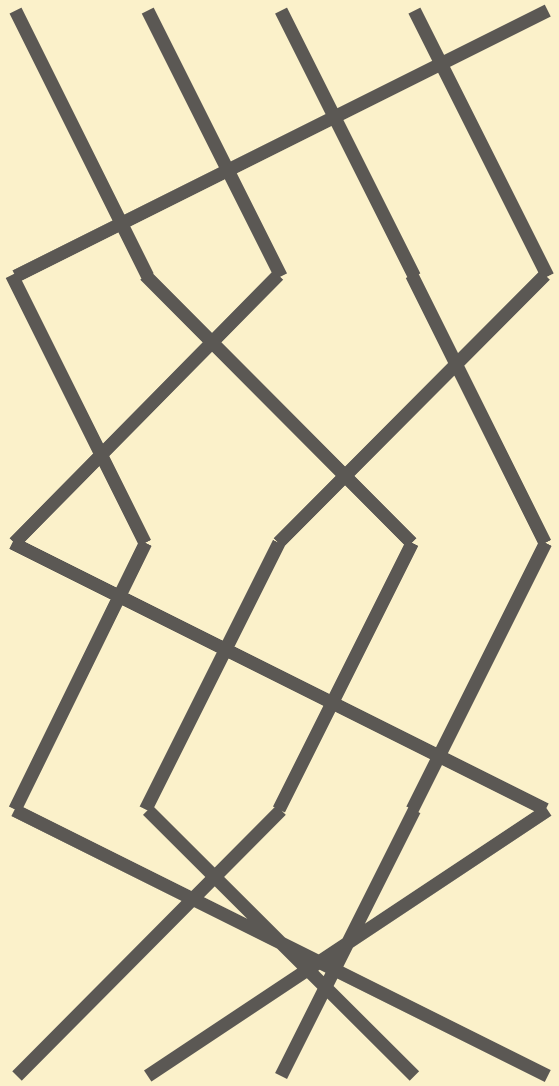

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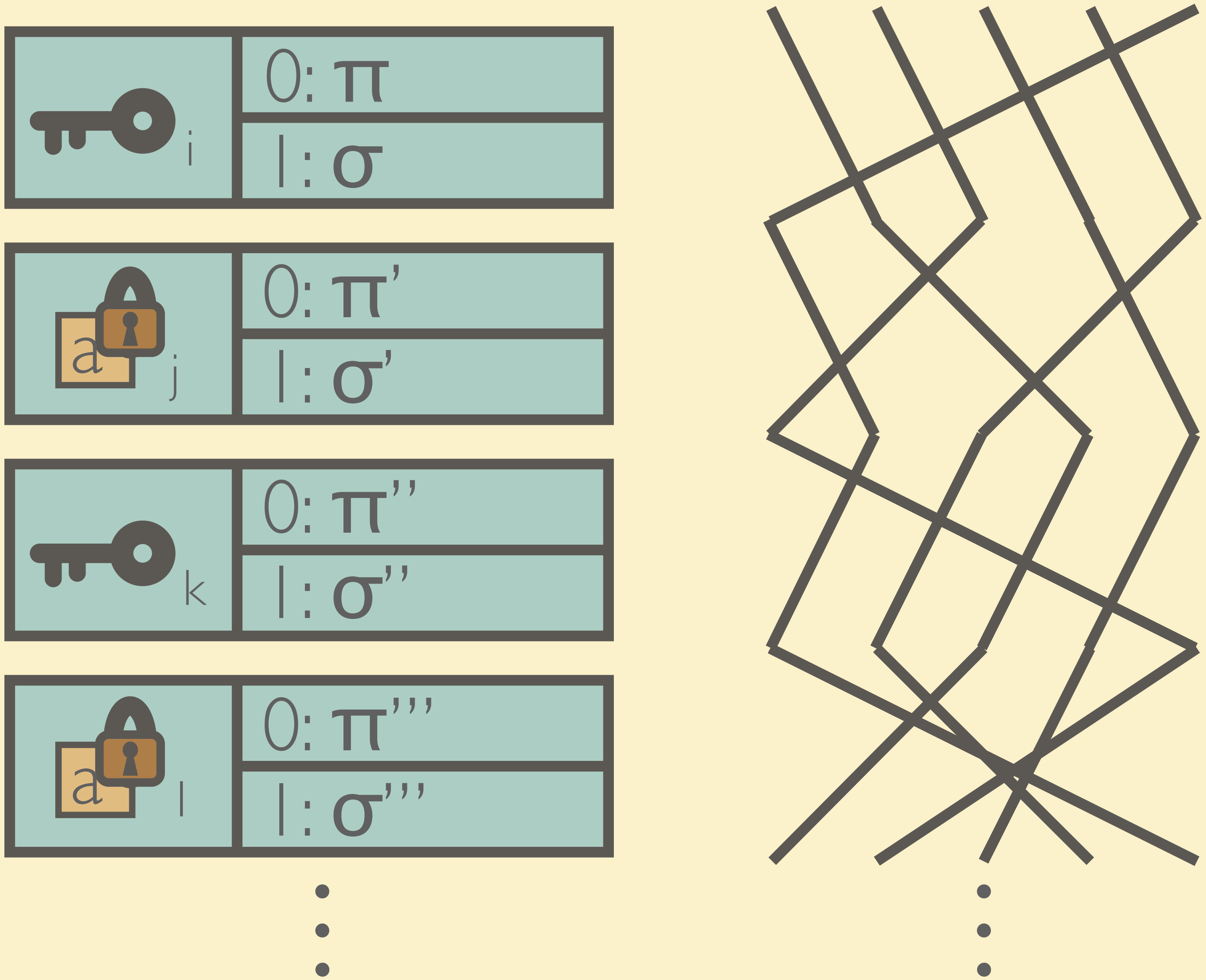

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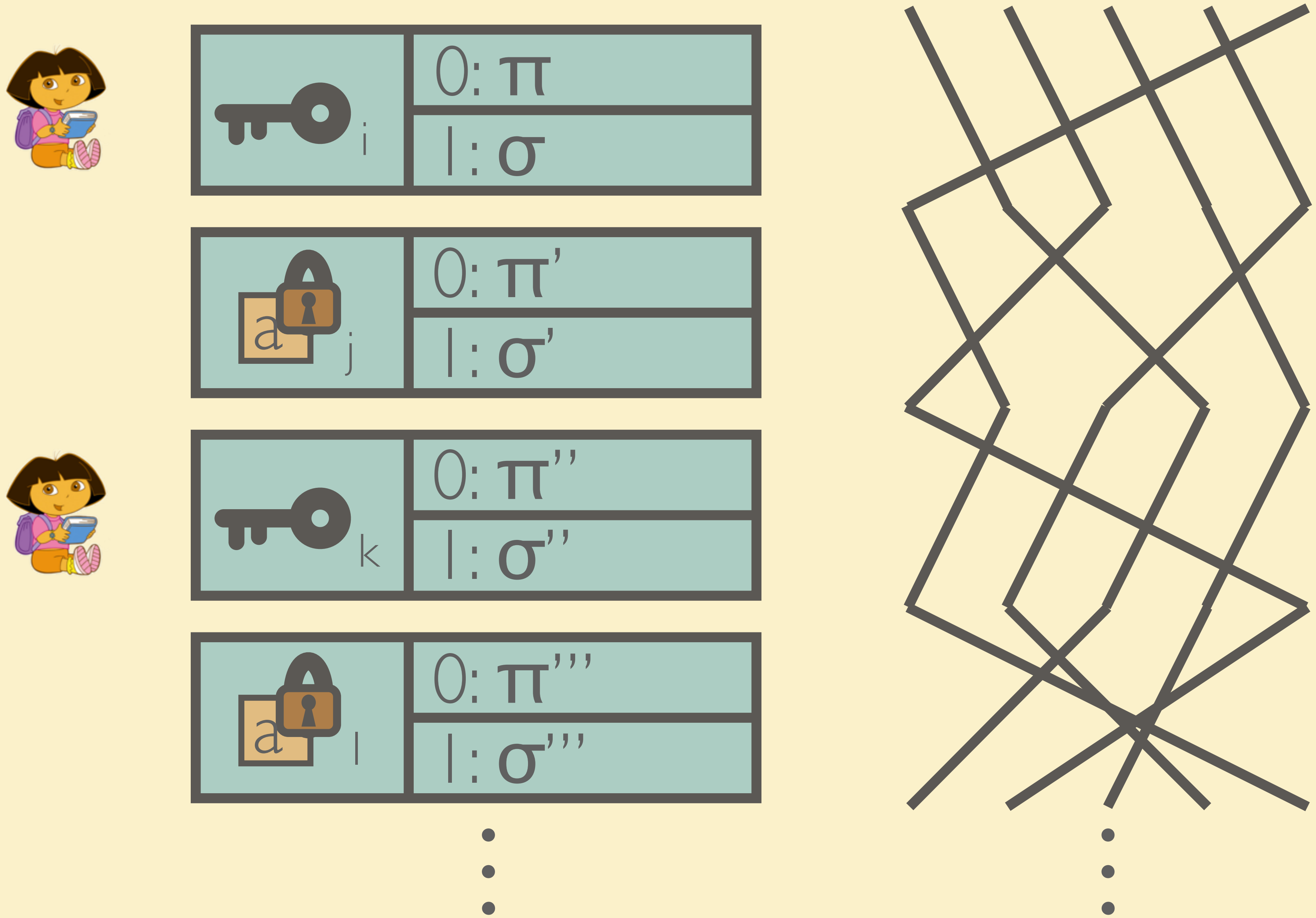

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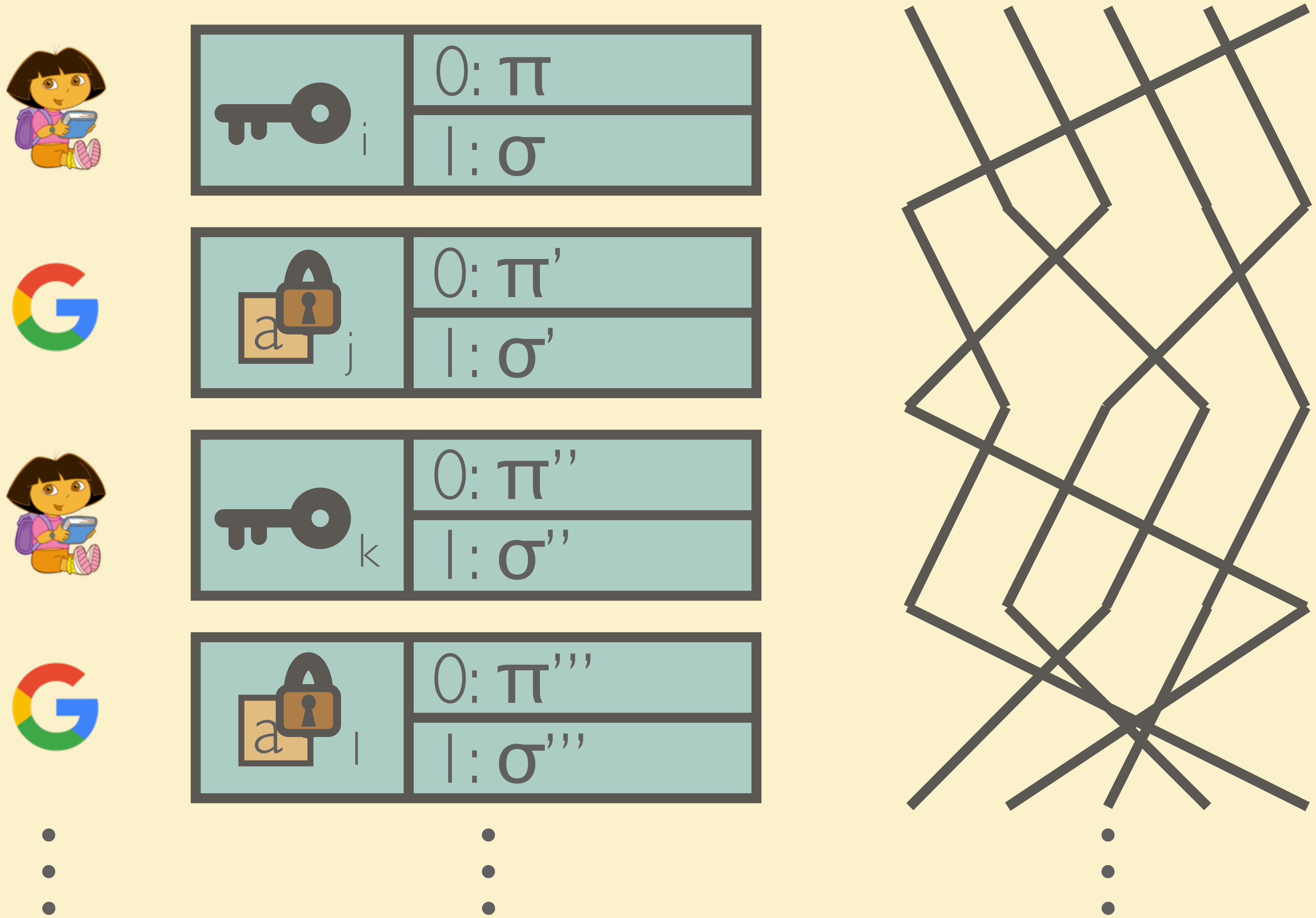

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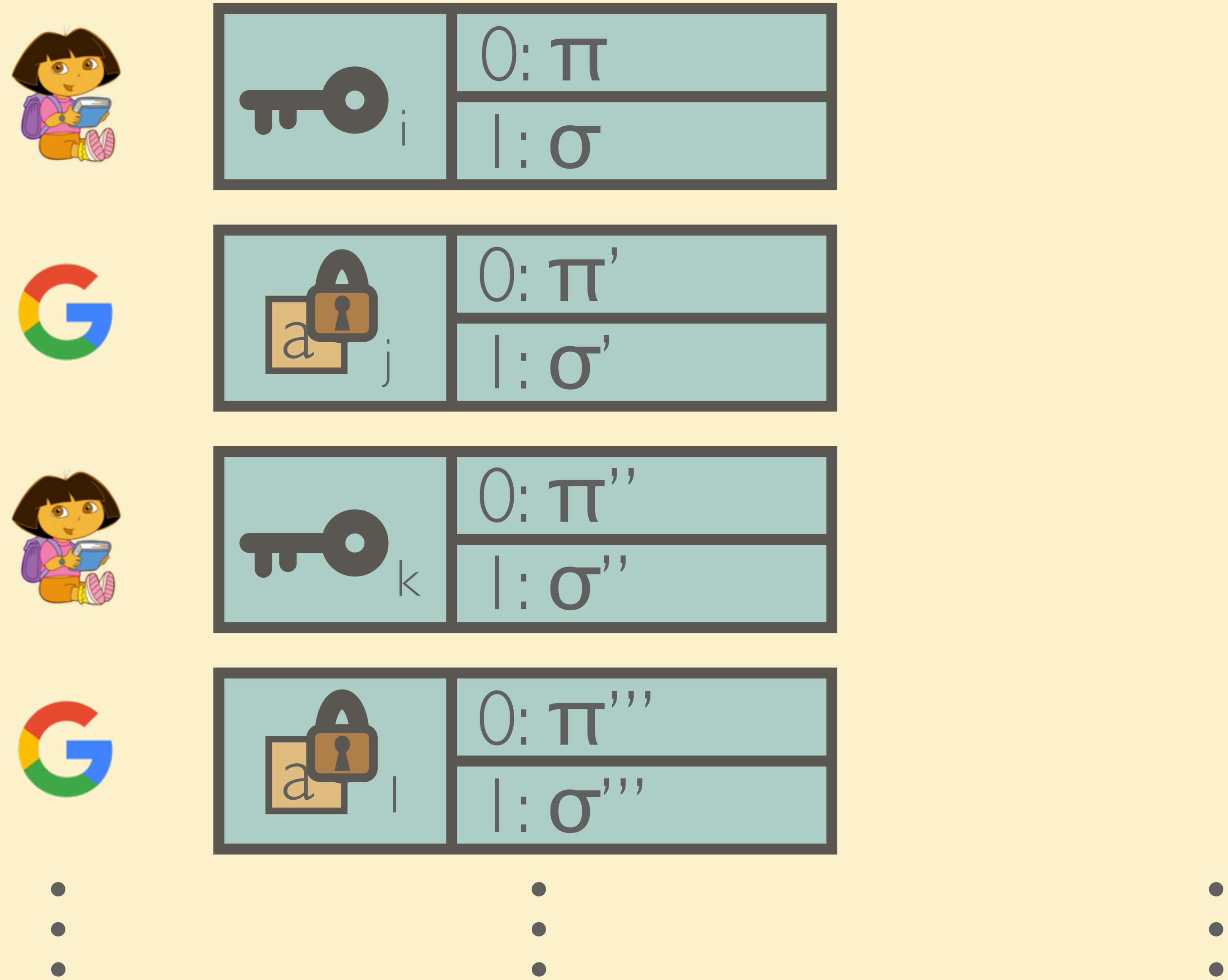

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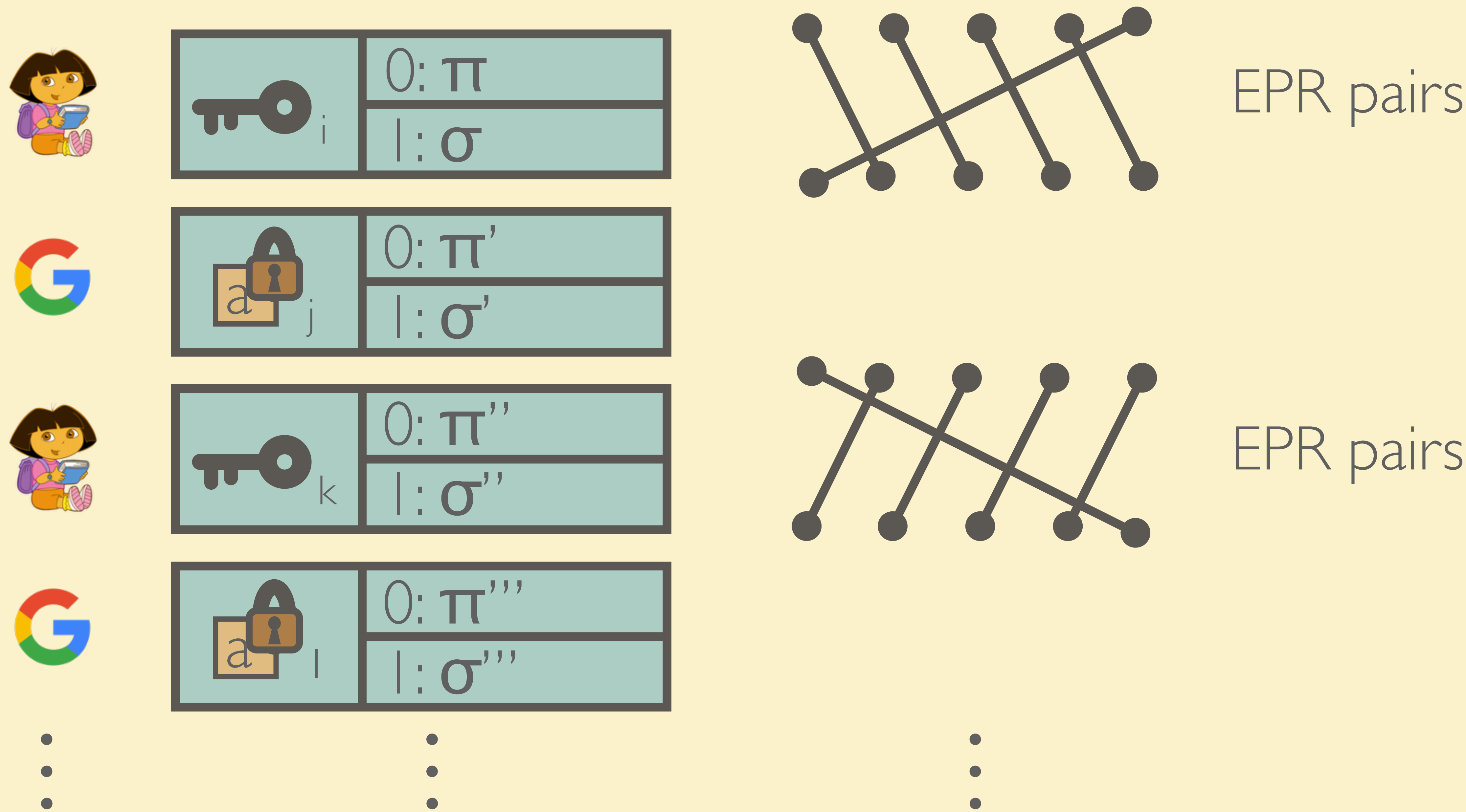

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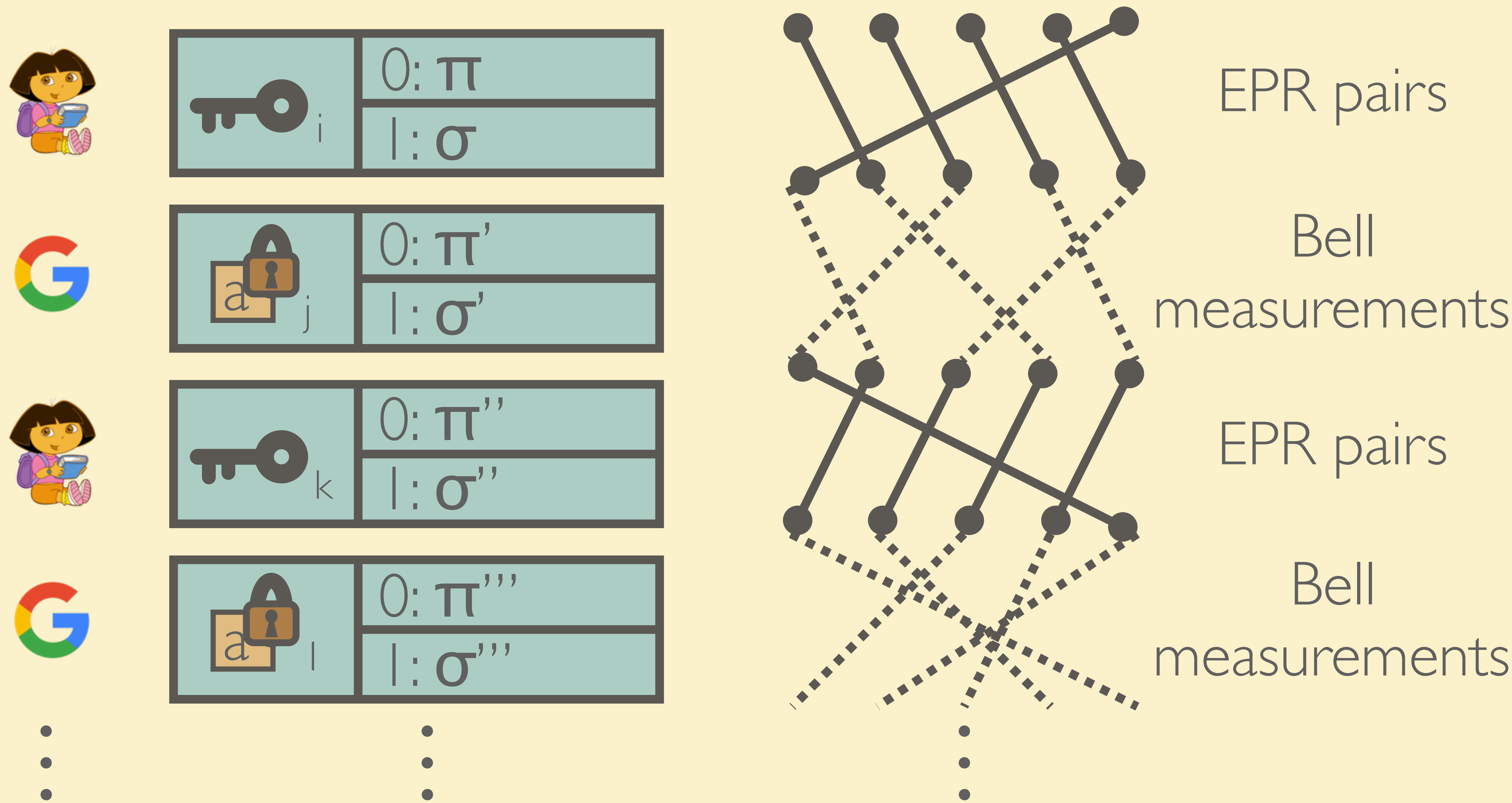

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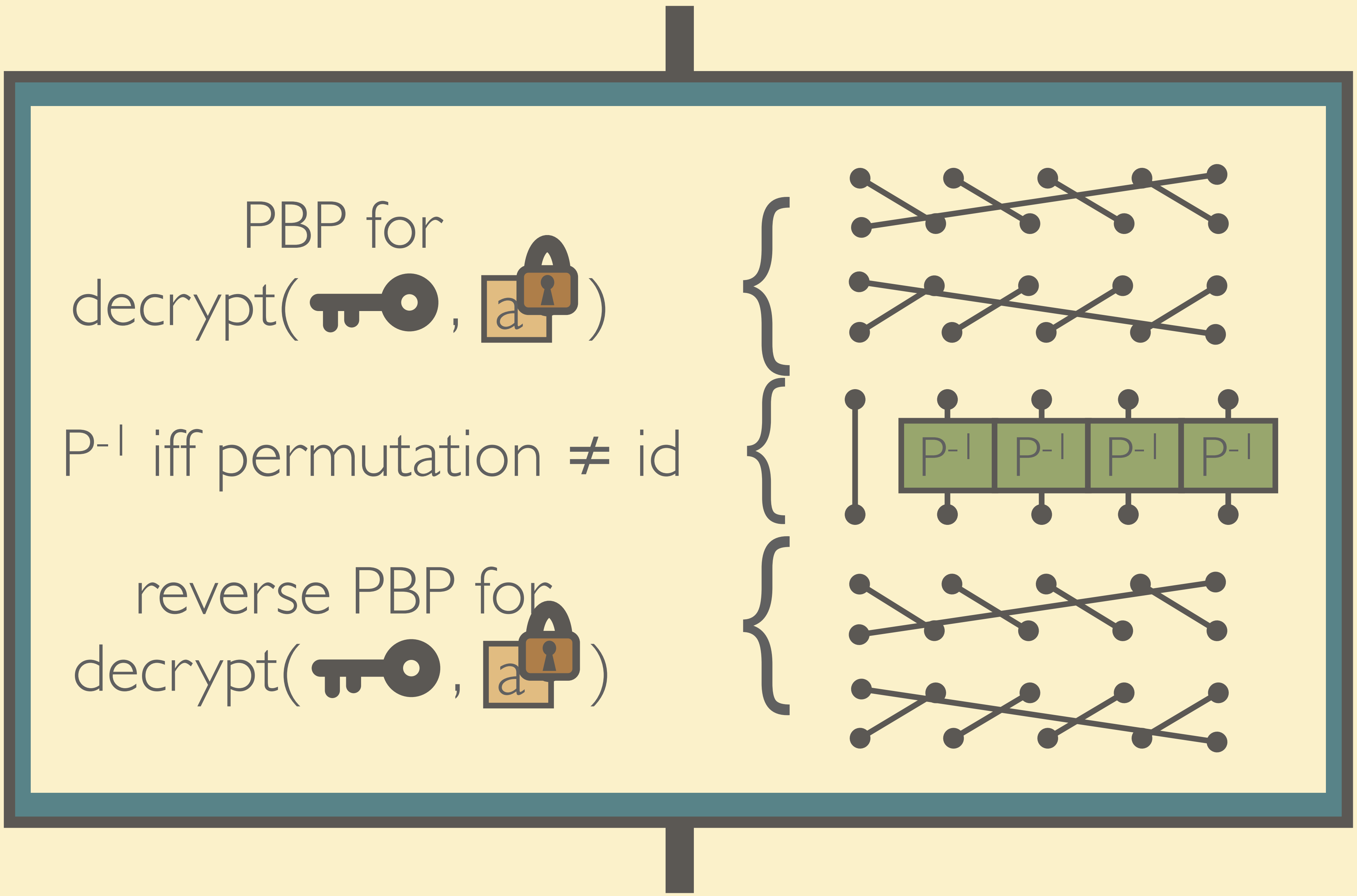

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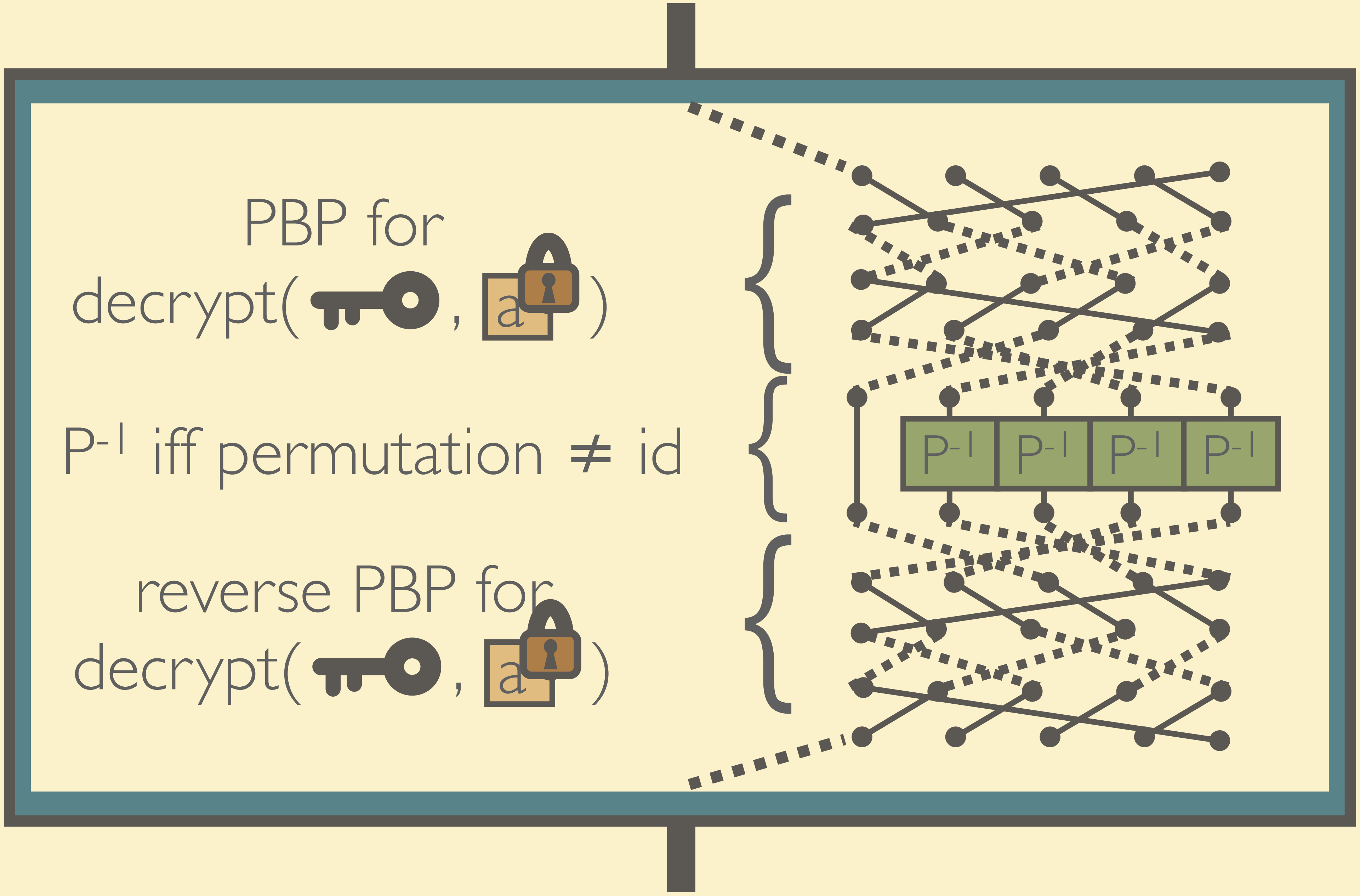

NEW SCHEME: OVERVIEW

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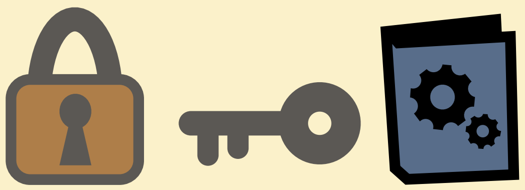
KEY GENERATION

NEW SCHEME: OVERVIEW

KEY GENERATION

- classical keys



NEW SCHEME: OVERVIEW

KEY GENERATION

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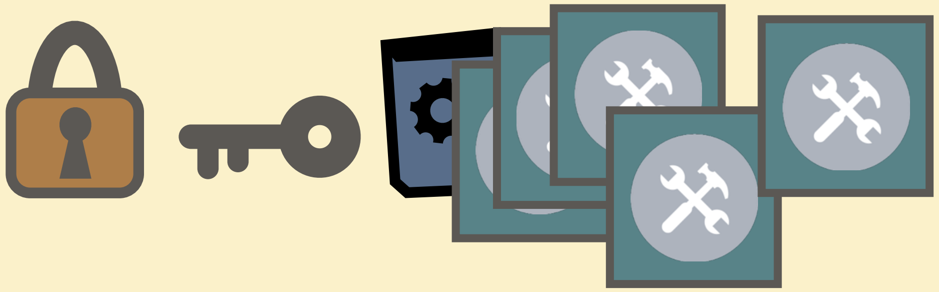

NEW SCHEME: OVERVIEW

KEY GENERATION

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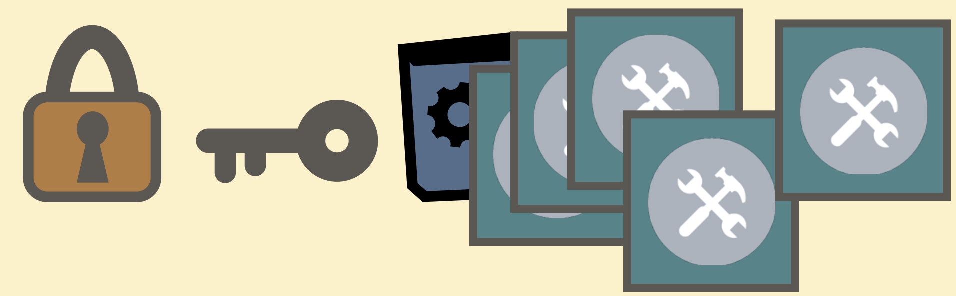
ENCRYPTION



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NEW SCHEME: OVERVIEW

KEY GENERATION

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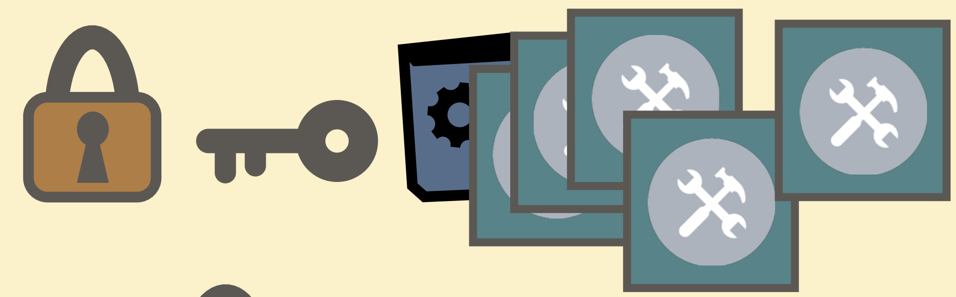
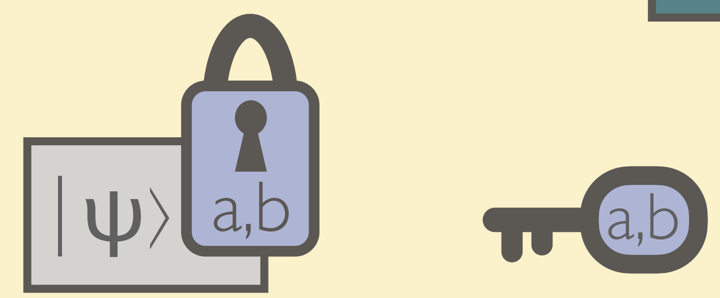


ENCRYPTION

- apply quantum one-time pad



NEW SCHEME: OVERVIEW

KEY GENERATION
- classical keys
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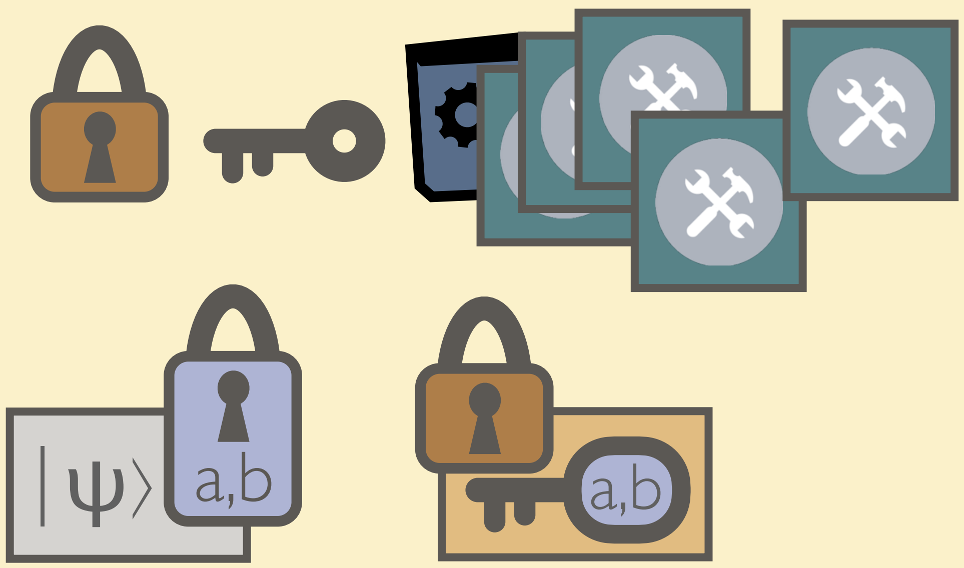
ENCRYPTION
- apply quantum one-time pad
- classically encrypt pad keys



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KEY GENERATION

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ENCRYPTION

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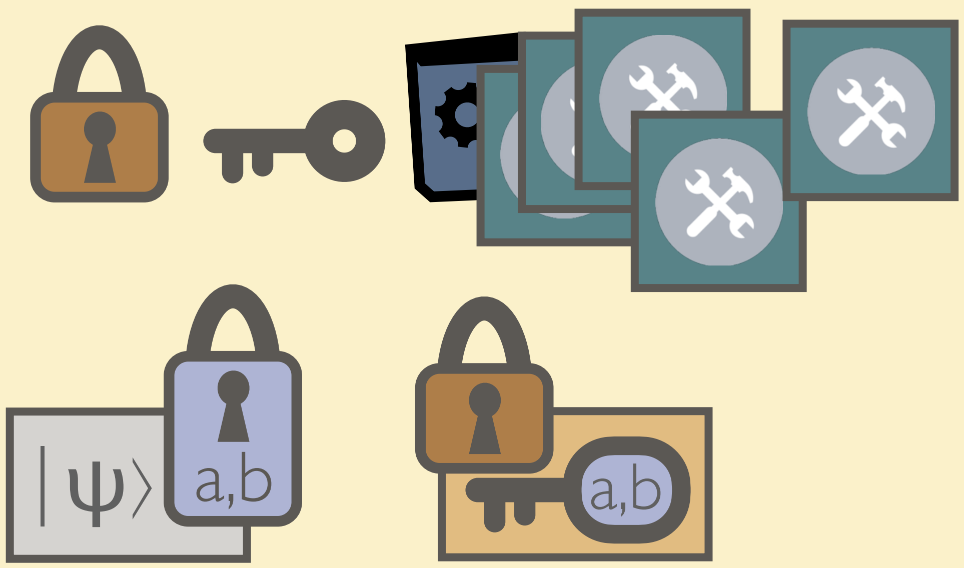
EVALUATION



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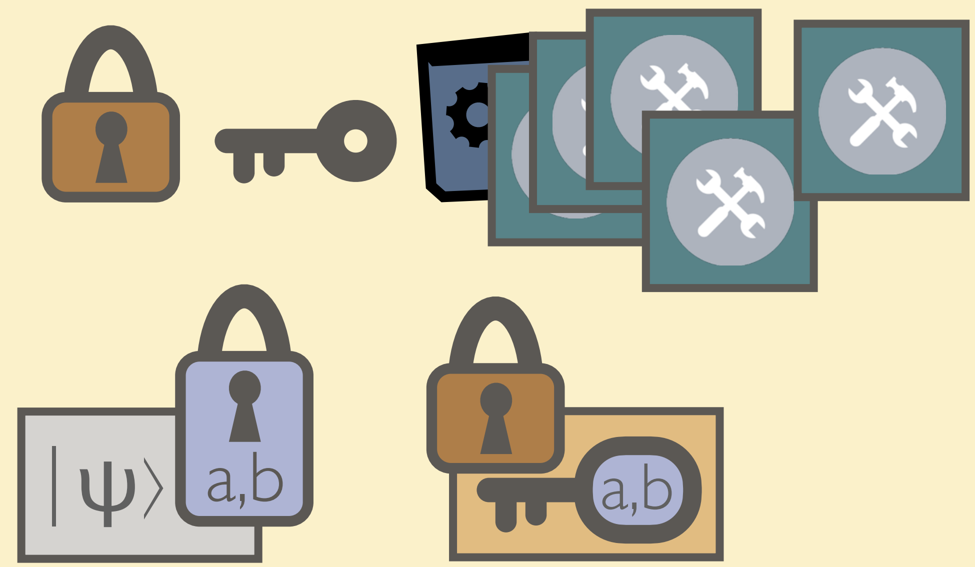


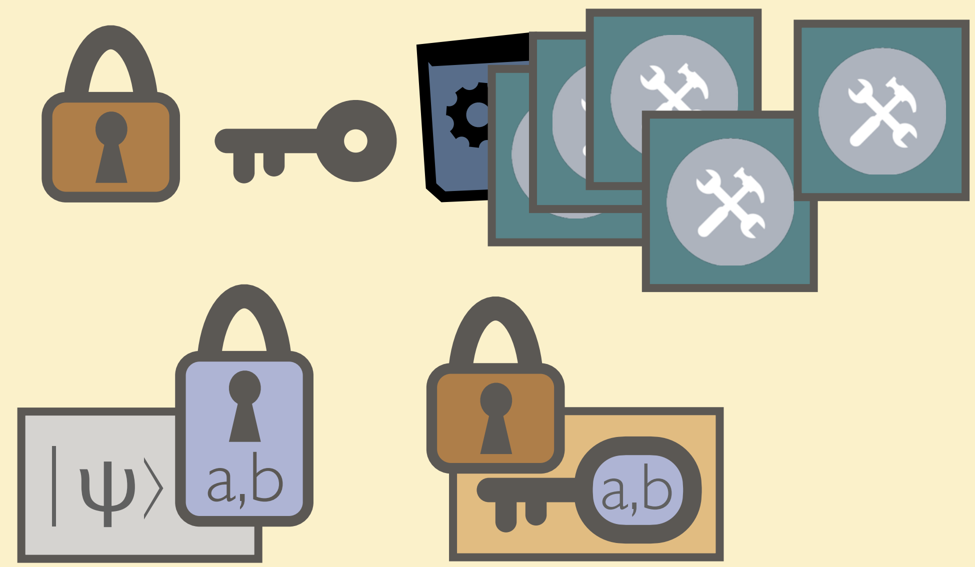
ENCRYPTION

- apply quantum one-time pad
- classically encrypt pad keys

EVALUATION

- after H / P / CNOT: classically update keys

NEW SCHEME: OVERVIEW



KEY GENERATION

- classical keys
- gadgets

ENCRYPTION

- apply quantum one-time pad
- classically encrypt pad keys

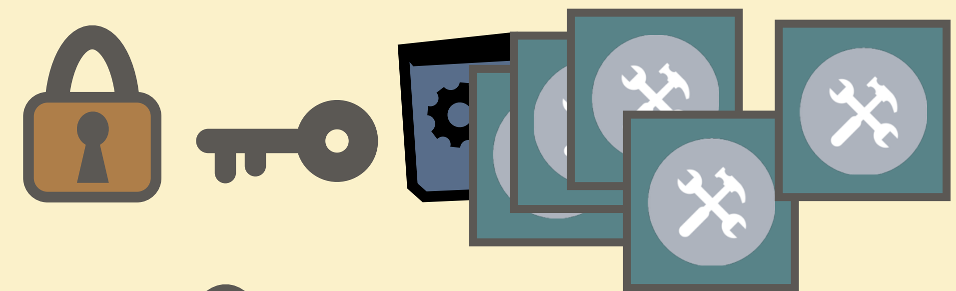
EVALUATION

- after H / P / CNOT: classically update keys
- after T : use 

NEW SCHEME: OVERVIEW

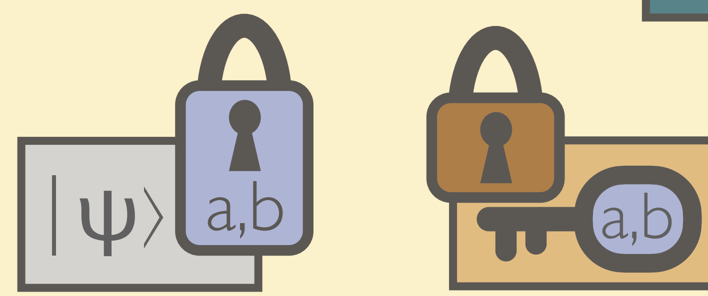
KEY GENERATION

- classical keys
- gadgets



ENCRYPTION

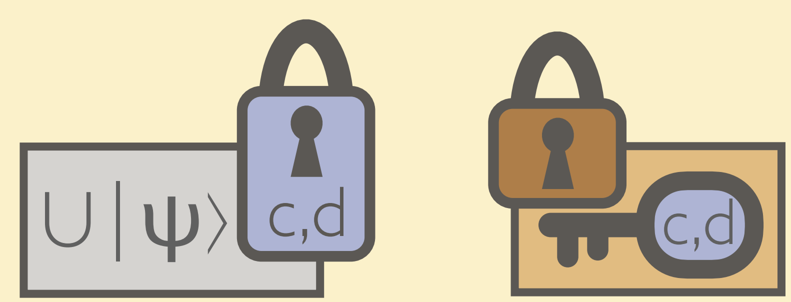
- apply quantum one-time pad
- classically encrypt pad keys



EVALUATION

- after H / P / CNOT: classically update keys
- after T: use 

DECRYPTION



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KEY GENERATION

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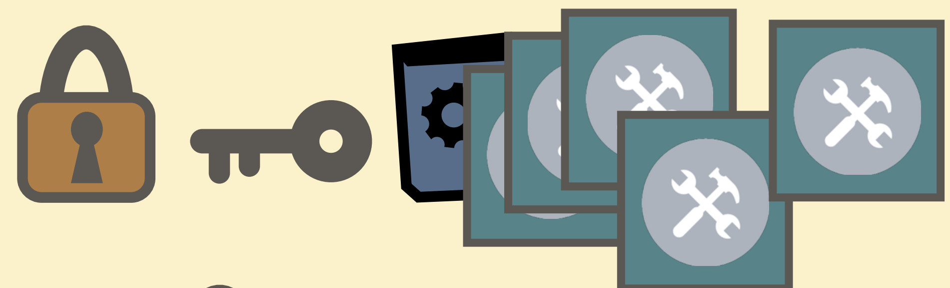

ENCRYPTION

- apply quantum one-time pad
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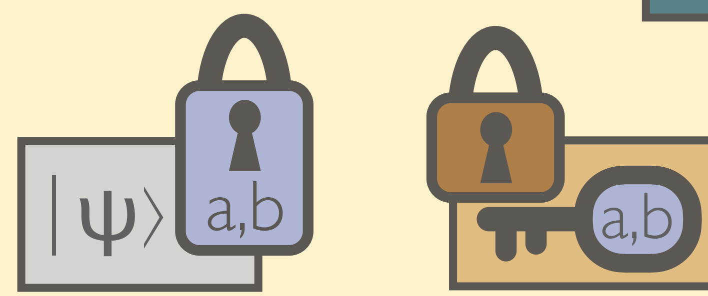


EVALUATION

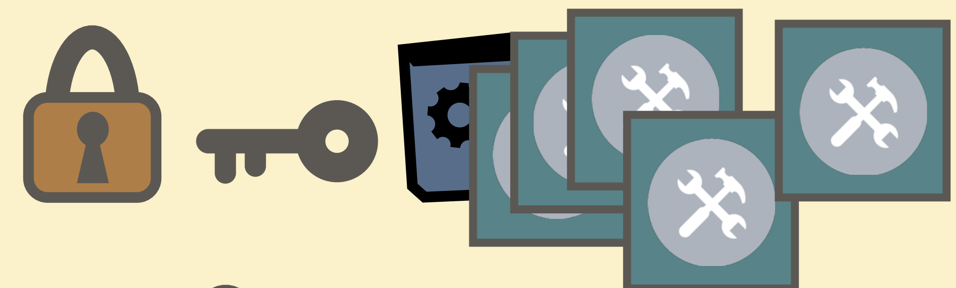
- after H / P / CNOT: classically update keys
- after T : use 

DECRYPTION

- classically decrypt pad keys

NEW SCHEME: OVERVIEW

KEY GENERATION



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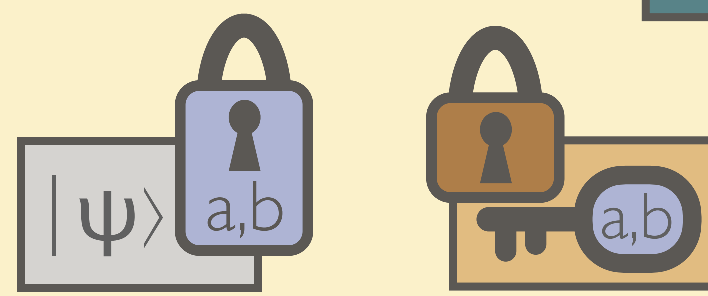
ENCRYPTION



- apply quantum one-time pad
- classically encrypt pad keys

EVALUATION

- after H / P / CNOT : classically update keys
- after T : use 

DECRYPTION

- classically decrypt pad keys
- remove quantum one-time pad

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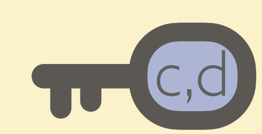

FUTURE WORK

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- non-leveled QFHE?

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- verifiable delegated quantum computation

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- verifiable delegated quantum computation
- quantum obfuscation?

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- …

 

THANK YOU!

QuSoft is hiring two principle investigators:
http://tinyurl.com/qusoft-job
Application deadline: 1 September 2016